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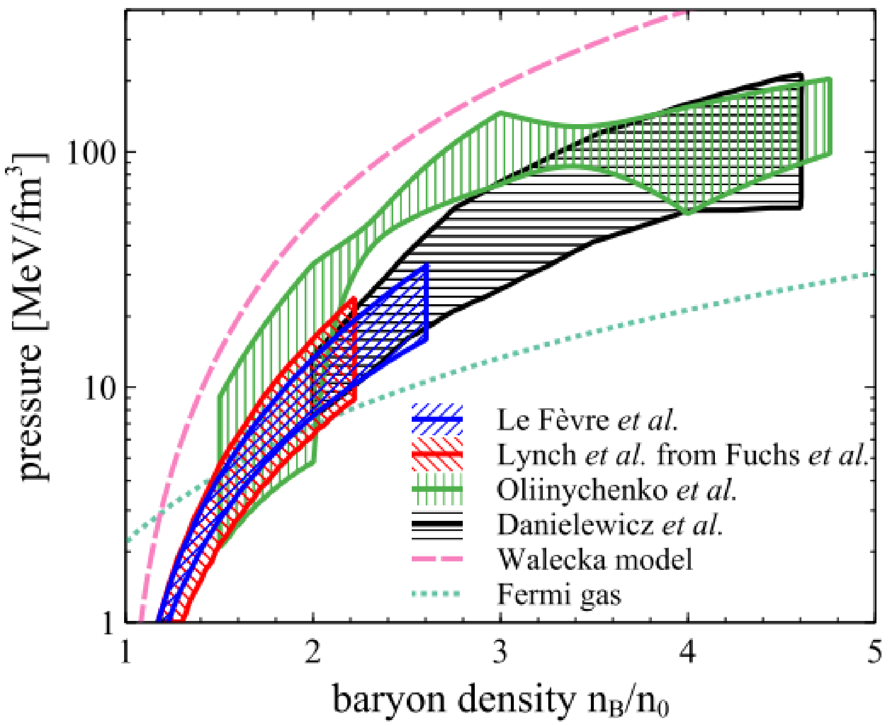
The in-medium nucleon-nucleon/Delta elastic cross section in intermediate-energy heavy-ion collisions

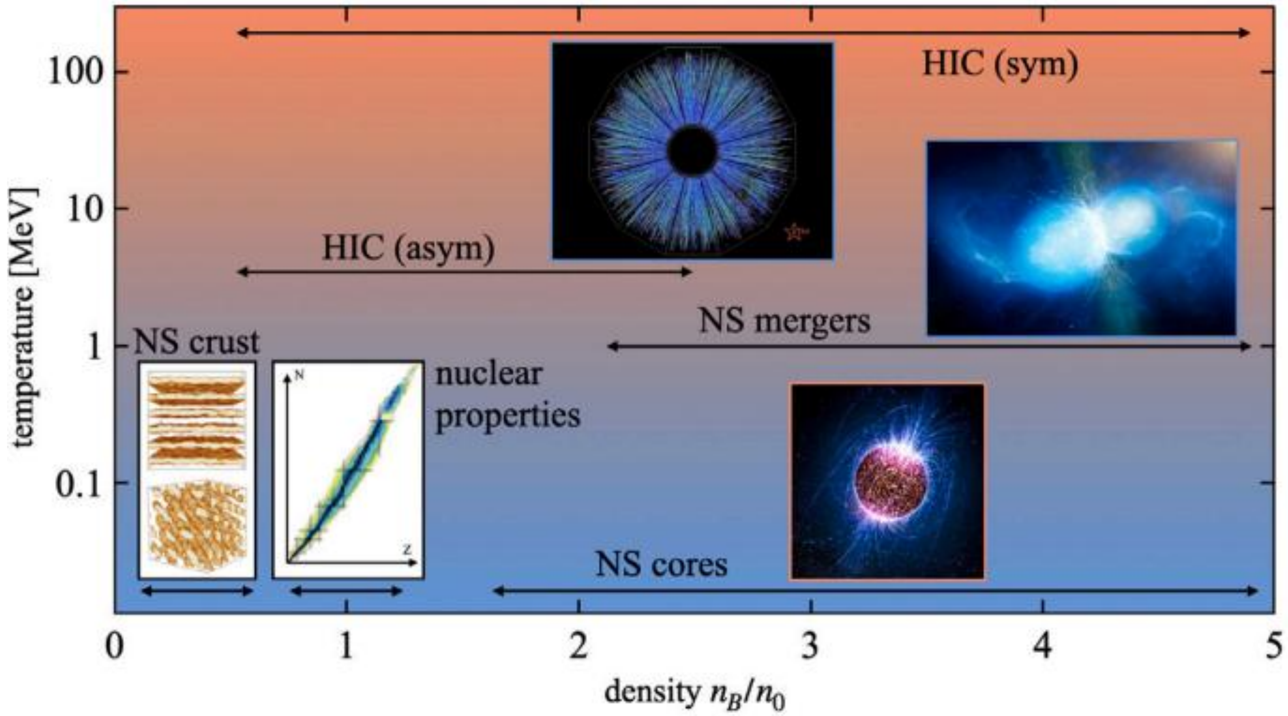
Pengcheng Li

Huzhou University

CONTENTS

EoS of nuclear matter





- The EoS is a fundamental property of nuclear matter, and determines the properties of nuclear matter at extreme densities.
- Below and near the saturation density ρ_0 with meaningful uncertainties, however, very large uncertainties at higher density.
- HICs at intermediate beam energies probe the widest ranges of baryon density, enabling studies of nuclear matter from a few tenths to about 5 times ρ_0.

A. Sorensen, *et al.*, Prog. Part. Nucl. Phys. 134 (2024) 104080; C. Y. Tsang, *et. al.*, Nat. Astron. 8, 328–336 (2024).

EoS of isospin asymmetric nuclear matter

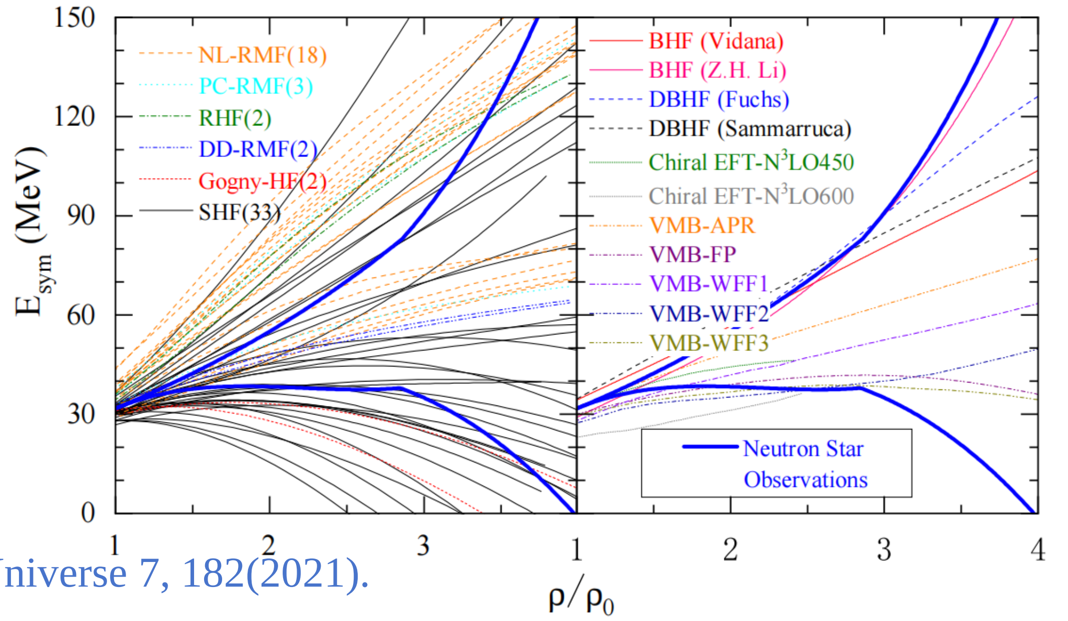
EoS of Isospin Asymmetric Nuclear Matter

$$E(\rho,\delta) = E(\rho,0) + E_{sym}(\rho)\delta^2 + O(\delta^4), \quad \delta = (\rho_n - \rho_p)/\rho$$

Symmetric Nuclear Matter (relatively well-determined)

Symmetry energy term (poorly known)

Isospin asymmetry

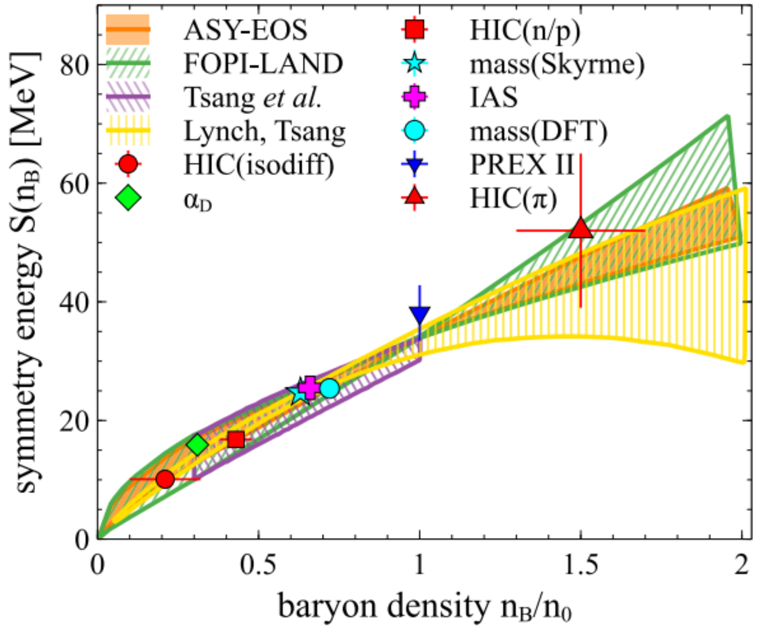


Nuclear Matter Symmetry Energy $E_{sym}(\rho) \equiv \dfrac{1}{2}\dfrac{\partial^2 E(\rho,\delta)}{\partial \delta^2}$

$$E_{sym}(\rho) = E_{sym}(\rho_0) + \frac{L}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right) + \frac{K_{sym}}{18}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 + \cdots\cdots$$

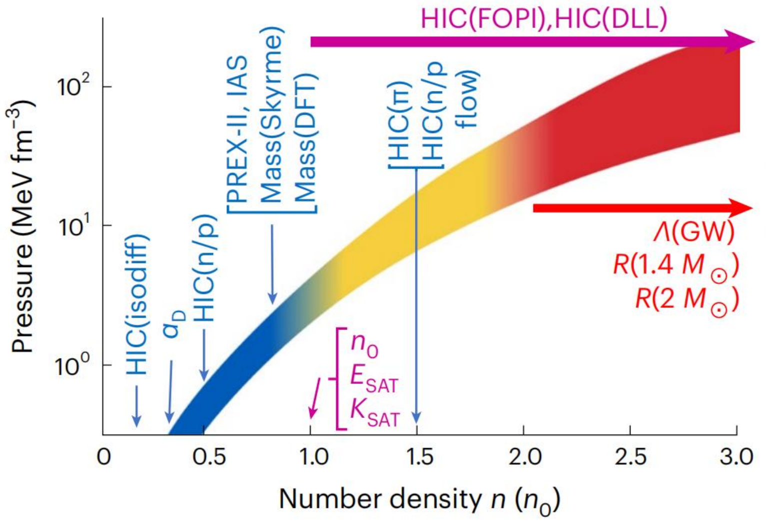
B. A. Li, *et. al.*, Universe 7, 182(2021).

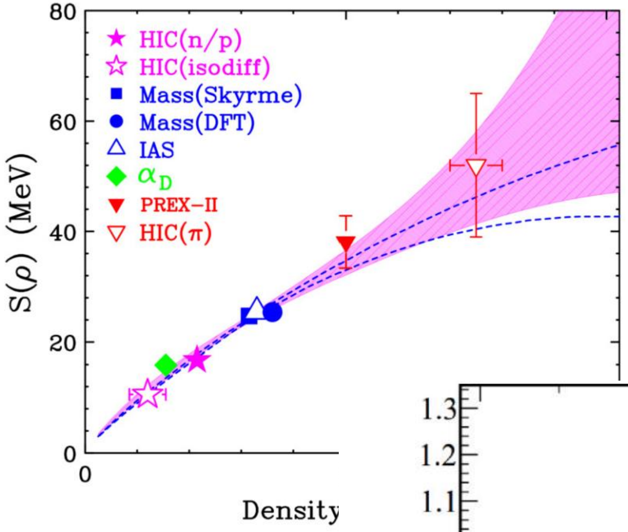
- The $E_{sym}(\rho)$ at supersaturation densities and the possible hadron–quark phase transition are among the most uncertain parts of the EoS of dense neutron-rich matter.
- With great efforts of both astrophysics and nuclear physics over the last two decades, rather consistent results on the characteristics of symmetry energy around ρ_0 have been extracted. Huge uncertainties remain at higher densities.

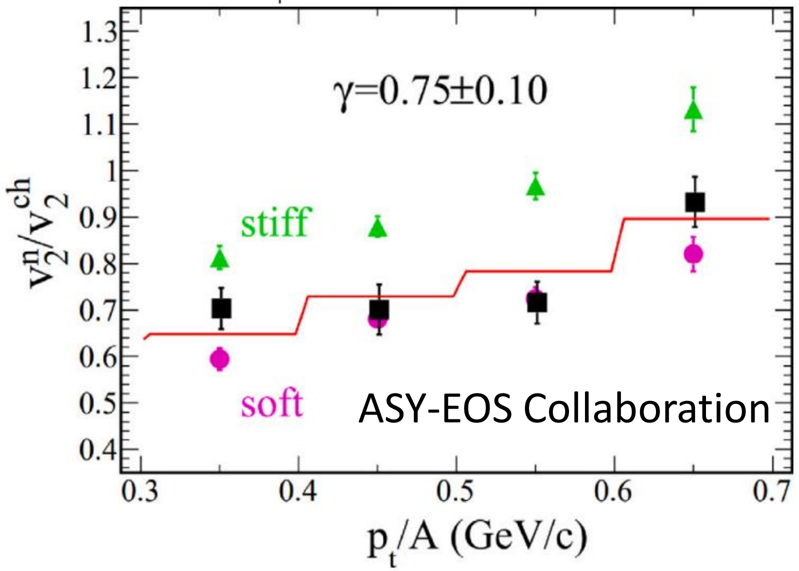


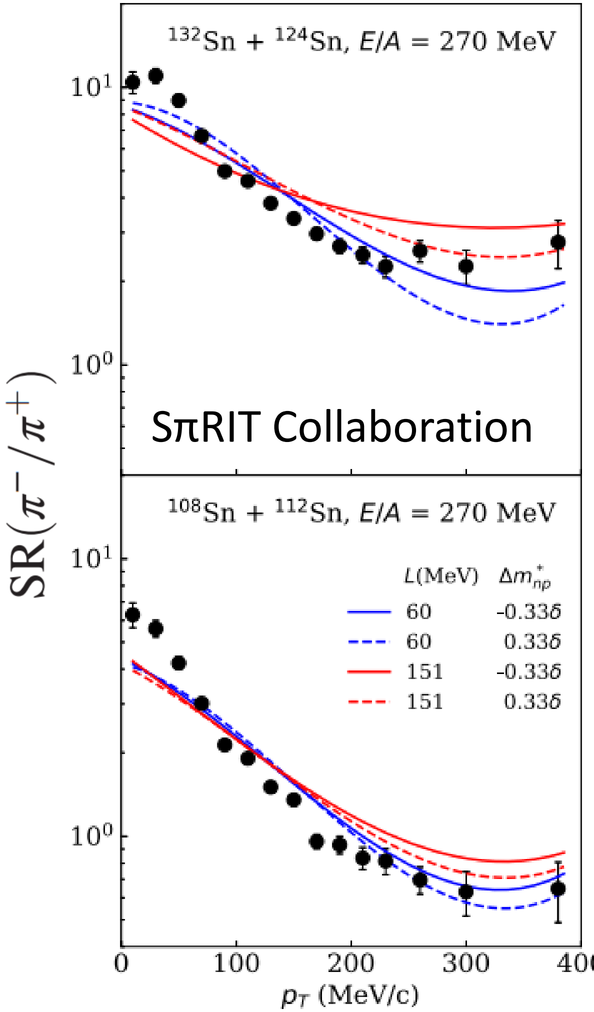
EoS of isospin asymmetric nuclear matter

- In HIC experiments, to quantitative constraints on the EoS, require comparisons of experimentally measured observables to results obtained in dynamic simulations.
- The symmetry energy contribution to the EoS can be studied by observables such as charged pion yields or neutron and proton flow.





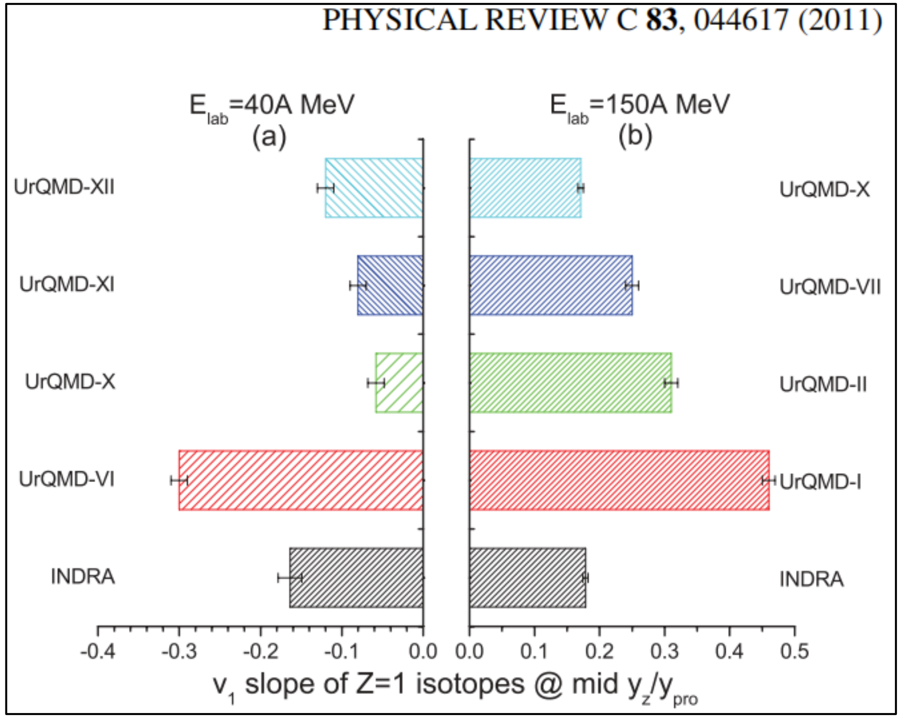


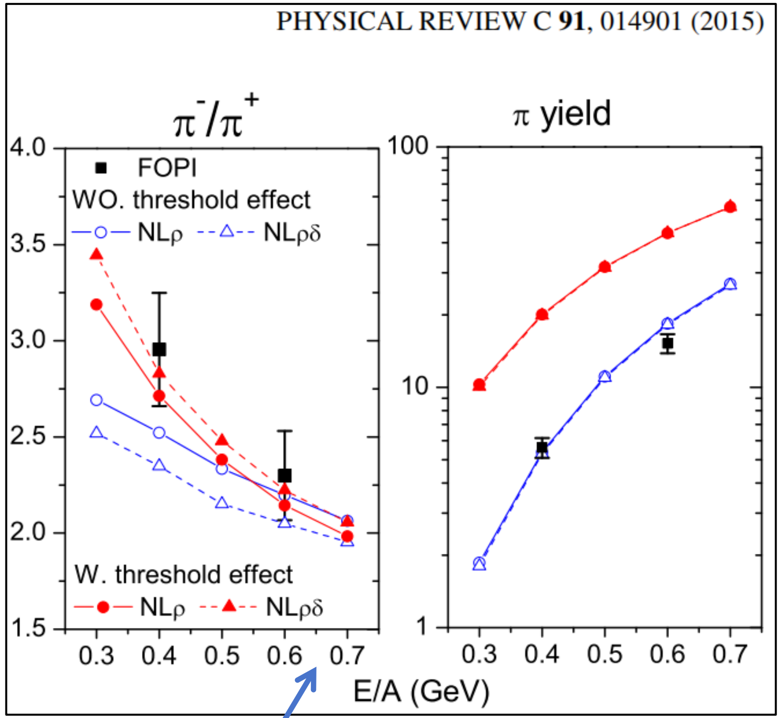


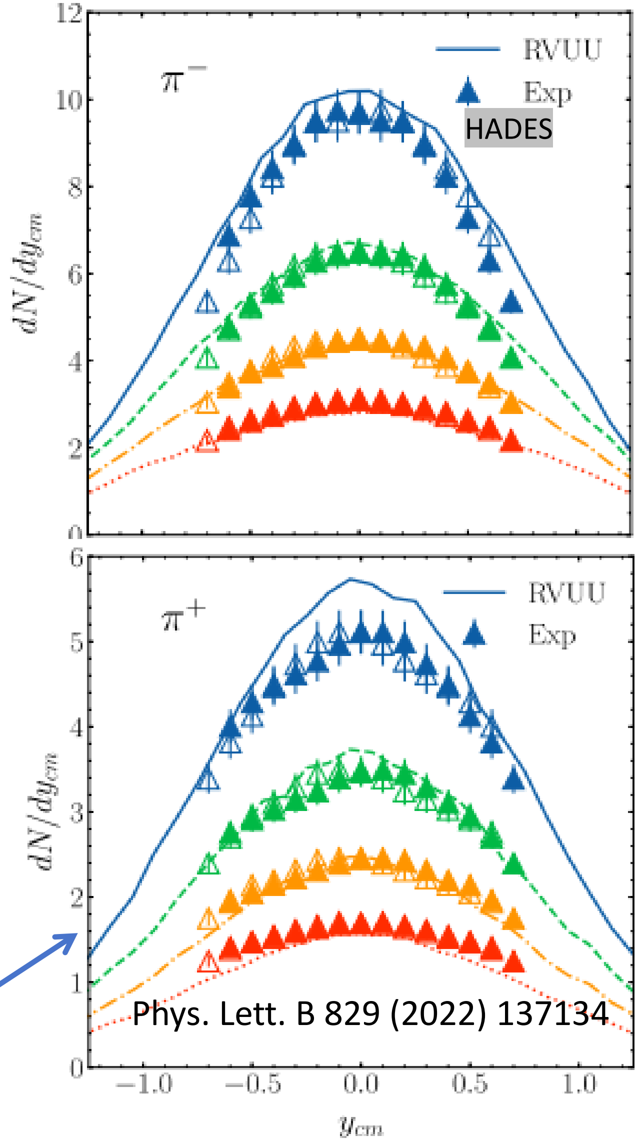
C. Y. Tsang, *et. al.*, Nat. Astron. 8, 328 (2024);
P. Russotto, *et al.*, Phys. Rev. C 94, 034608 (2016);
J. Estee, *et al.*, Phys. Rev. Lett. 126, 162701(2021);
S. Huth, *et al.*, Nature 606, 276 (2022);
W. G. Lynch, M. B. Tsang, Phys. Lett. B 830, 137098(2022).

In-medium effects of cross section on observables

- The mean-field and collisions are two essential inputs of the hadronic transport model, and the isospin observables will be influenced.







- In-medium threshold effect enhances both the total pion yield and the pion ratio.
- With the medium dependence of the *Δ* resonance production cross section, one can obtain a good description of the rapidity distributions of charged-pion for various centrality bins.

In-medium nucleon-nucleon elastic cross section

RBUU transport theory：

EQUILIBRIUM AND NONEQUILIBRIUM FORMALISMS MADE UNIFIED

Kuang-chao CHOU, Zhao-bin SU, Bai-lin HAO and Lu YU

Institute of Theoretical Physics, Academia Sinica, P.O. Box 2735, Beijing, China

ANNALS OF PHYSICS **83**, 491–529 (1974)

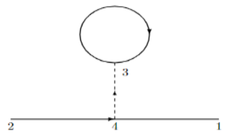
A Theory of Highly Condensed Matter*

J. D. WALECKA

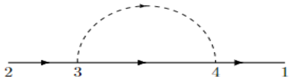
Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California 94305

Received September 17, 1973

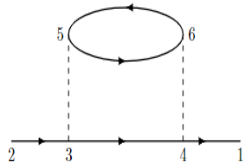
Feynman diagram



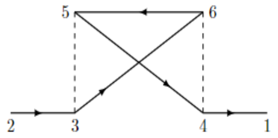
Hartree term



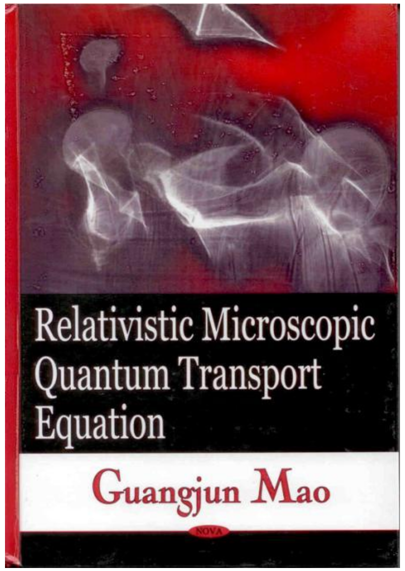
Fock term



Born term



Exchange term



BUU equation

$$\left(\partial_t + \frac{\mathbf{p}}{m}\cdot\nabla_{\mathbf{R}} - \nabla_{\mathbf{R}} U(R,t)\cdot\nabla_{\mathbf{p}}\right) f(\mathbf{p},\mathbf{R},t) \equiv I(f)$$

RBUU equation

$$\left\{\left[\partial_X^\mu - \sum\nolimits_{HF}^{\mu\nu}(X,p,t)\partial_\nu^{\bar{p}} - \partial_{\bar{p}}^\nu \,\mathrm{Re}\sum\nolimits_F^\mu(X,p,t)\partial_\nu^X\right]\frac{\bar{p}_\mu}{M^*_{\mathrm{Re}}} + \partial_\nu^X\left[\sum\nolimits_H(X) + \mathrm{Re}\sum\nolimits_F(X,p,t)\right]\partial_p^\nu - \partial_p^\nu \,\mathrm{Re}\sum\nolimits_F(X,p,t)\partial_\nu^X\right\}\times\frac{M}{E(\bar{p})} f(X,\vec{\bar{p}})$$
$$= \frac{1}{2}\int \frac{d\vec{p}_2}{(2\pi)^3}\frac{d\vec{p}_3}{(2\pi)^3}\frac{d\vec{p}_4}{(2\pi)^3}(2\pi)^4\delta(\vec{\bar{p}} + \vec{p}_2 - \vec{p}_3 - \vec{p}_4)\delta(E(\bar{p}) + E(p_2) - E(p_3) - E(p_4))$$
$$\times W_{el,in}(\bar{p}, p_2, p_3, p_4)(F^2_{el,in} - F^1_{el,in})$$

RBUU equation derivation of nucleon distribution function

1、 Green function of nucleon

$$iG_{12} = \left\langle T\left[\exp\left(-i\oint dx H_I(x)\right)\psi(1)\bar{\psi}(2)\right]\right\rangle,$$

$$G_{12} = \begin{pmatrix} G_{12}^{--} & G_{12}^{-+} \\ G_{12}^{+-} & G_{12}^{++} \end{pmatrix}.$$

2、Series expansion of Green's function

$$iG_{12} = iG_{12}^0 + \oint dx_3 \oint dx_4 G_{14}^0 \Sigma(4,3) iG_{32},$$

$$\Sigma(4,3) = \Sigma_{\text{HF}}(4,3) + \Sigma_{\text{Born}}(4,3).$$

3、Introducing the Dirac field operator

$$G_{01}^{-1} = i\gamma\cdot\partial_{x1} - M.$$

4、Operate it to Green's function

$$G_{01}^{-1} iG_{12} = i\delta(1,2) + \oint d3 \sum(1,3) iG_{32}.$$

5、Transform it into momentum space and simplify

$$[\gamma_\mu K^\mu(X,P) - M(X,P)]iG^{-+}(X,P) = F_C(X,P).$$

6、Define the single-particle phase space distribution function

$$\frac{1}{4}\text{tr}\left[iG^{-+}(x,p)\right] = -\frac{\pi m^*}{E^*}\delta(p_0 - E^*(p)) f(\mathbf{x},\mathbf{p},\tau).$$

7、Then we get the RBUU equation of the nucleon

$$\{[\partial_x^\mu - \Sigma_{\text{HF}}^{\mu\nu}(x,p)\partial_\nu^p - \partial_p^\nu \Sigma_{\text{HF}}^\mu(x,p)\partial_\nu^x]p_\mu + m^*[\partial_\nu^x \Sigma_{\text{HF}}^S(x,p)\partial_p^\nu - \partial_p^\nu \Sigma_{\text{HF}}^S(x,p)\partial_\nu^x]\}\frac{f(\mathbf{x},\mathbf{p},\tau)}{E^*} = C(x,p)$$

$$\Sigma_{\text{HF}}^{\mu\nu}(x,p) = \partial_x^\mu \Sigma_{\text{HF}}^\nu(x,p) - \partial_x^\nu \Sigma_{\text{HF}}^\mu(x,p)$$
$$p^\mu(x,p) = P^\mu - \Sigma_{\text{HF}}^\mu(x,p)$$
$$m^*(x,p) = M_N + \Sigma_{\text{HF}}^S(x,p)$$

G. Mao, *et al.*, Phys. Rev. C 49, 3137-3146 (1994).

RBUU equation derivation of nucleon distribution function

The collision term can be expressed as:

$$C(x,p)=\frac{1}{2}\int\frac{d^3p_2}{(2\pi)^3}\int\frac{d^3p_3}{(2\pi)^3}\int\frac{d^3p_4}{(2\pi)^3}(2\pi)^4\delta^{(4)}\left(p+p_2-p_3-p_4\right)W\left(p,p_2,p_3,p_4\right)\left(F_2-F_1\right),$$

$$W\left(p,p_2,p_3,p_4\right)=G_1\left(p,p_2,p_3,p_4\right)+G_2\left(p,p_2,p_3,p_4\right)+p_3\leftrightarrow p_4,$$

$$\begin{aligned}F_1&=f(\mathbf{x},\mathbf{p},\tau)f(\mathbf{x},\mathbf{p}_2,\tau)[1-f_\Delta(\mathbf{x},\mathbf{p}_3,\tau)][1-f(\mathbf{x},\mathbf{p}_4,\tau)],\\F_2&=[1-f(\mathbf{x},\mathbf{p},\tau)][1-f(\mathbf{x},\mathbf{p}_2,\tau)]f_\Delta(\mathbf{x},\mathbf{p}_3,\tau)f(\mathbf{x},\mathbf{p}_4,\tau),\end{aligned}$$

The relationship between the scattering cross section and the collision term is:

$$\int\frac{d^3p_3}{(2\pi)^3}\int\frac{d^3p_4}{(2\pi)^3}(2\pi)^4\delta^{(4)}\left(p+p_2-p_3-p_4\right)W\left(p,p_2,p_3,p_4\right)=\int v\sigma(s,t)d\Omega.$$

Therefore the collision term can be expressed as:

$$C(x,p)=\frac{1}{2}\int\frac{d^3p_2}{(2\pi)^3}v\sigma(s,t)\left(F_2-F_1\right)d\Omega.$$

G. Mao, *et al.*, Phys. Rev. C 49, 3137-3146 (1994).

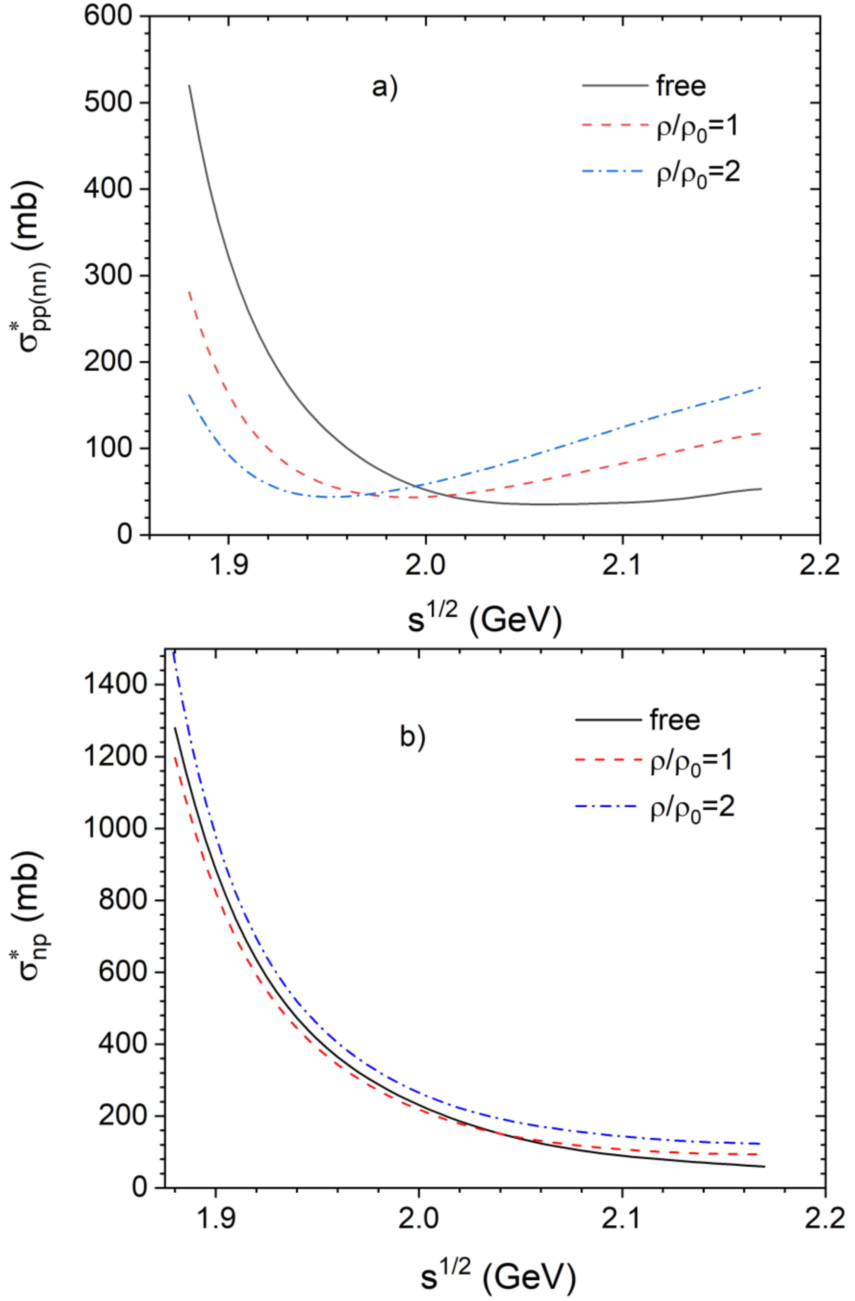
Isospin and density dependence of nucleon-nucleon elastic cross section

L_I is the interaction Lagrangian density of nucleons coupled to σ, ω, ρ, and π mesons and reads as

$$L_I = g_\sigma \overline{\Psi}\Psi\sigma - g_\omega \overline{\Psi}\gamma_\mu \Psi \omega^\mu + g_\pi \overline{\Psi}\gamma_\mu \gamma_5 \boldsymbol{\tau}\cdot\Psi \partial^\mu \boldsymbol{\pi}$$

$$-\frac{1}{2} g_\rho \overline{\Psi}\gamma_\mu \boldsymbol{\tau}\cdot\Psi \boldsymbol{\rho}^\mu,$$

- The medium correction of nucleon-nucleon scattering cross sections is isospin dependent.
- The ρ meson field plays a dominant role in the isospin dependence of the nucleon-nucleon elastic cross sections.
- σ^*_{np} depends on the baryon density weakly, while $\sigma^*_{pp(nn)}$ depends on the baryon density significently. Which is due to the different effects of the medium correction of nucleon mass and ρ meson mass on σ^*_{np} and $\sigma^*_{pp(nn)}$, respectively.

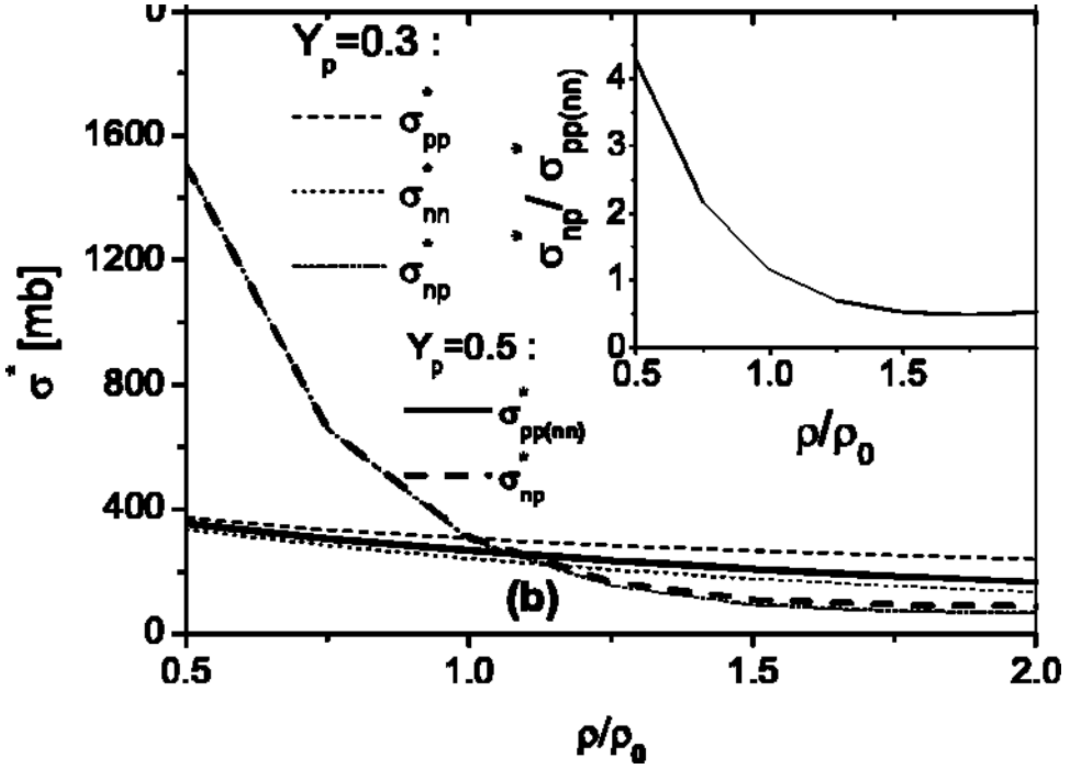


Q. F. Li, Z. X. Li, G. J. Mao, Phys. Rev. C 62, 014606(2000).

Density and temperature dependence of nucleon-nucleon elastic cross section

The scalar-isovector δ meson is further introduced into the effective Lagrangian to calculate the in-medium two-body scattering elastic cross sections

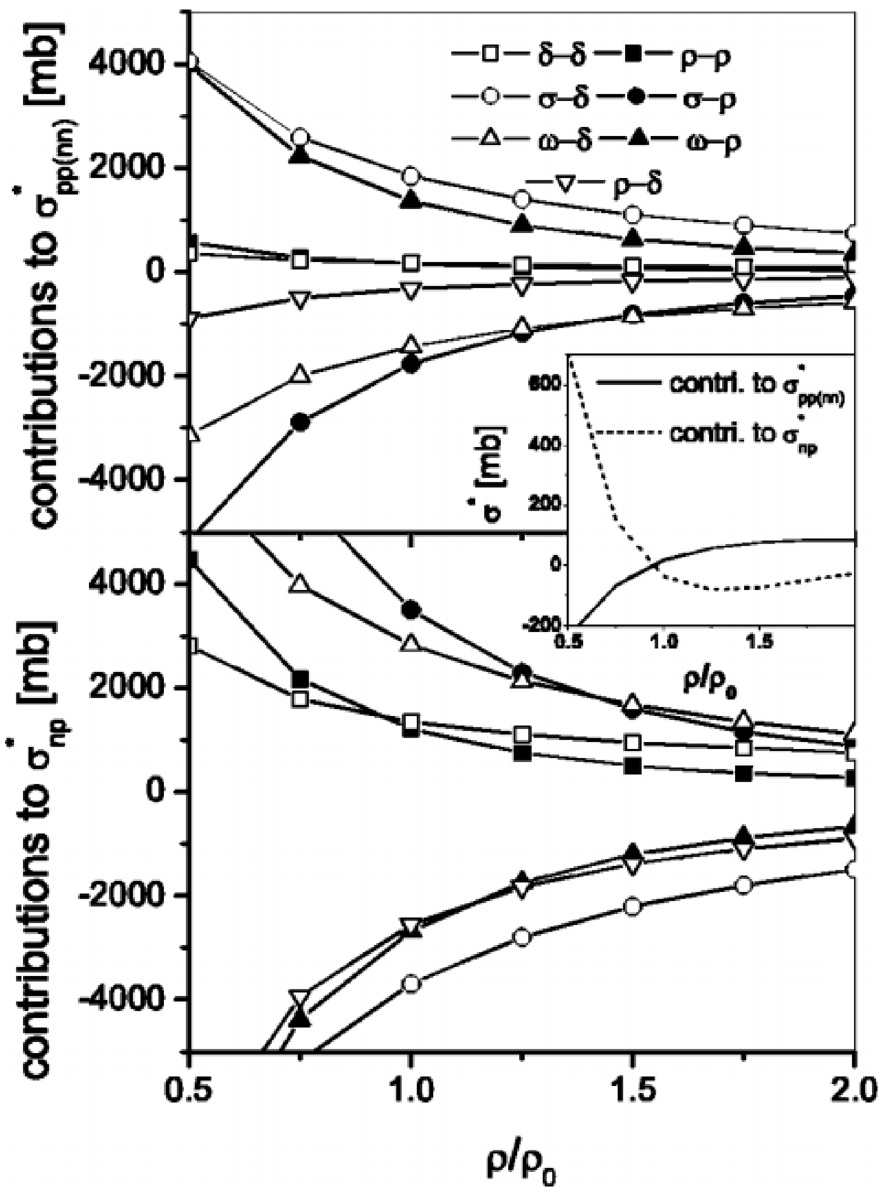
$$
\begin{aligned}
L = {} & \bar{\Psi}[i\gamma_\mu\partial^\mu - M_N]\Psi + \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \tfrac{1}{4}F_{\mu\nu}\cdot F^{\mu\nu} + \tfrac{1}{2}\partial_\mu\boldsymbol{\delta}\partial^\mu\boldsymbol{\delta} \\
& - \tfrac{1}{4}L_{\mu\nu}\cdot L^{\mu\nu} - \tfrac{1}{2}m_\sigma^2\sigma^2 + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \tfrac{1}{2}m_\delta^2\boldsymbol{\delta}^2 + \tfrac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu \\
& + g_\sigma\bar{\Psi}\Psi\sigma - g_\omega\bar{\Psi}\gamma_\mu\Psi\omega^\mu + g_\delta\bar{\Psi}\boldsymbol{\tau}\cdot\Psi\boldsymbol{\delta} \\
& - \tfrac{1}{2}g_\rho\bar{\Psi}\gamma_\mu\boldsymbol{\tau}\cdot\Psi\boldsymbol{\rho}^\mu,
\end{aligned}
$$

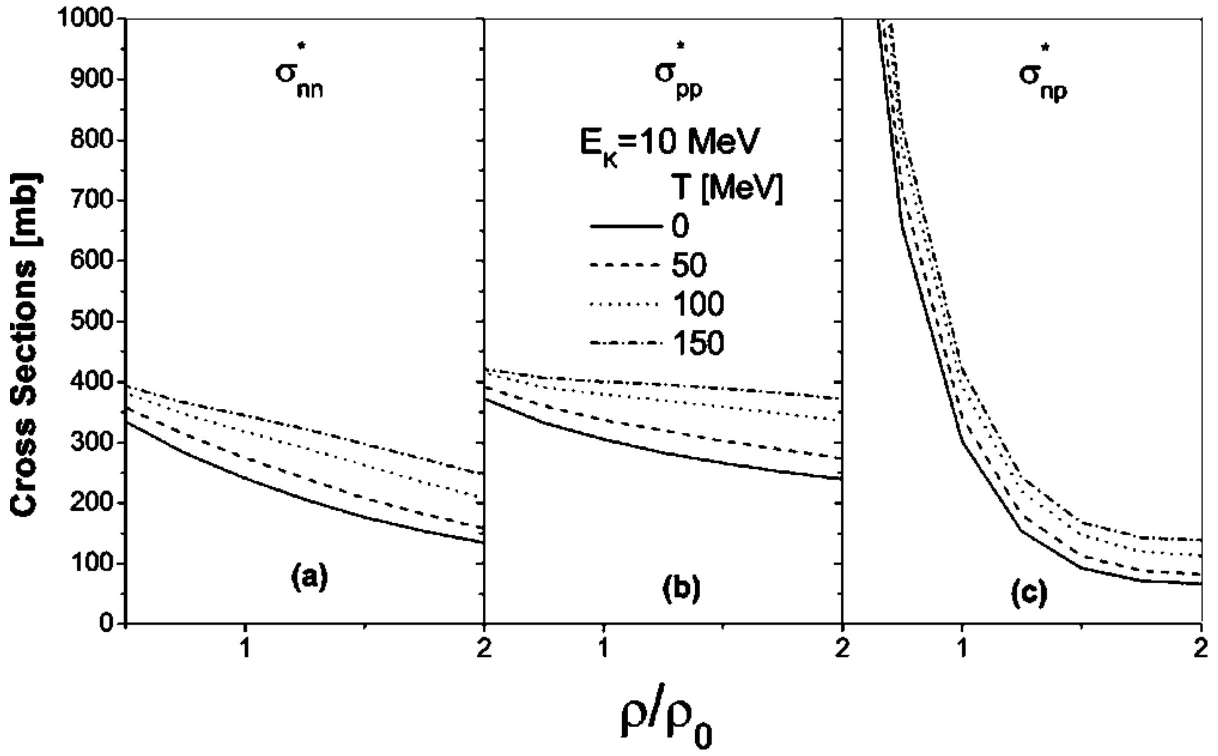


At low densities, the σ^*_{np} is about three to four times larger than $\sigma^*_{pp(nn)}$, at densities higher than the normal density the isospin effect is almost washed out.

Q. F. Li, Z. X. Li, E. G. Zhao, Phys. Rev. C 69 (2004) 017601

Density and temperature dependence of nucleon-nucleon elastic cross section





- The isospin effect on the density dependence of the in-medium nucleon elastic cross section is dominantly contributed by the isovector δ and ρ mesons.
- The temperature effect of nuclear medium on σ^*_{np} and $\sigma^*_{pp(nn)}$ is weaker compared with the density effect.

In-medium nucleon-nucleon elastic cross section in hadronic transport model

In HICs, the parameterized in-medium correction factor on *NN* elastic cross sections is commonly used for simplicity. In general, this correction factor is **density-** and/or **momentum-**, as well as **isospin**-dependent, for example：

- Density dependent and energy independent.
  - G. D. Westfall *et al.*, Phys. Rev. Lett. 71, 1986 (1993).

$$\sigma^* = (1 - \eta\frac{\rho}{\rho_0})\sigma^{free}$$

- BUU& RBUU: $\eta \approx 0.2$
  - D. Klakow *et al*, Phys. Rev. C 48, 1982 (1993).
  - T. Gaitanos *et al*, Phys. Lett. B609, 241 (2005).
- ImQMD: $\eta = 0.2 \rightarrow E_{lab} < 150A$ MeV, $\eta = 0 \rightarrow 150 < E_{lab} < 200A$ MeV, $\eta = -0.2 \rightarrow 200 < E_{lab} < 400A$ MeV,
  - Y. X. Zhang *et al.*, Phys. Rev. C. 75, 034615 (2007).

- pBUU: Density, energy and isospin dependent.
  - P. Danielewicz, Acta. Phys. Pol. B 33, 45 (2002); D. D. S. Coupland *et al.*, Phys. Rev. C. 84, 054603 (2011).

$$\sigma^* = y\rho^{-2/3}\tanh(\sigma_{free}/\sigma_0).$$

- IBUU: Density, isospin and momentum dependent.
  - V. R. Pandharipande *et al.*, Phys. Rev. C. 45, 791 (1991).
  - B. A. Li *et al.*, Phys. Rev. C. 72, 064611 (2005).

$$\sigma^* = \left(\frac{\mu^*_{NN}}{\mu_{NN}}\right)^2 \sigma^{free},$$

TMEP collaboration, *et al.*, Prog. Part. Nucl. Phys. 125 (2022) 103962.

In-medium effects on nucleon-nucleon elastic cross section

Density- and momentum-dependend

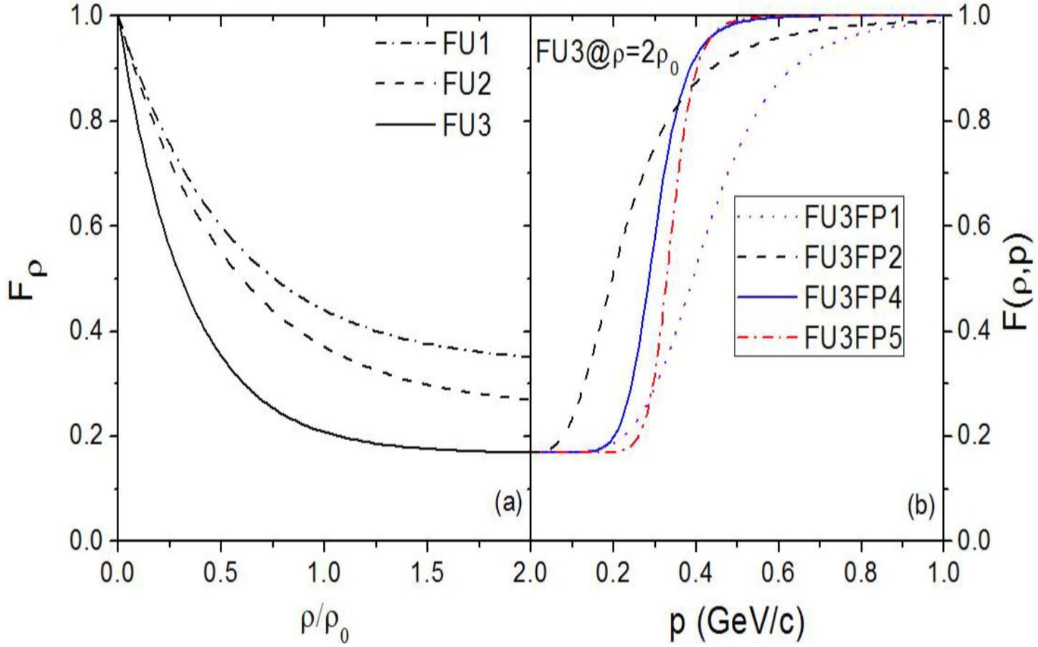
$$\sigma_{\text{tot}}^* = \sigma_{\text{in}} + \sigma_{\text{el}}^* = \sigma_{\text{in}} + F(\rho,p)\sigma_{\text{el}},$$

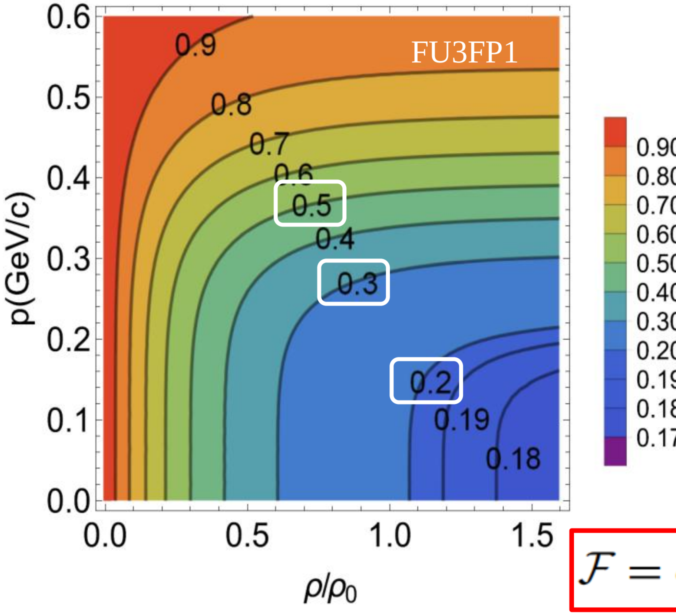
$$F(\rho,p) = \begin{cases} f_0 & p_{NN} > 1\ \text{GeV}/c, \\ \frac{F_\rho - f_0}{1+(p_{NN}/p_0)^\kappa} + f_0 & p_{NN} \leqslant 1\ \text{GeV}/c, \end{cases}$$

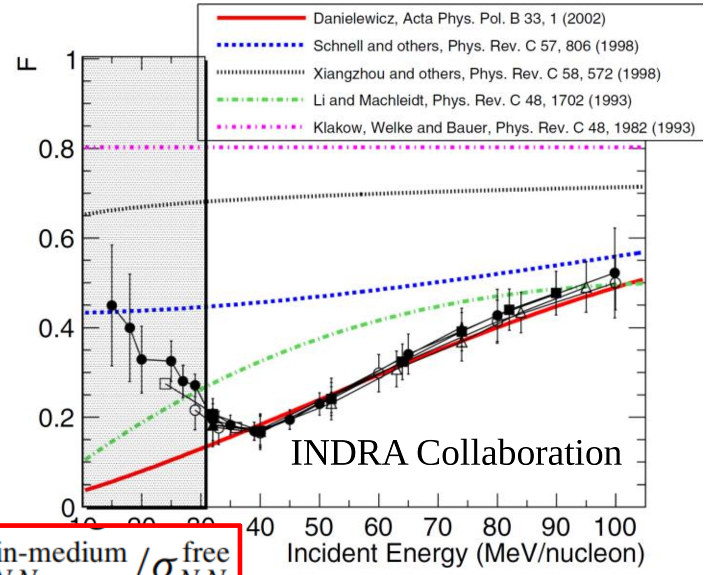
$$F_\rho = \lambda + (1-\lambda)\exp\left[-\frac{\rho}{\zeta\rho_0}\right]$$

Set	λ	ζ
FU1	1/3	0.54568
FU2	1/4	0.54568
FU3	1/6	1/3

Set	f_0	p_0 (GeV/c)	κ
FP1	1	0.425	5
FP2	1	0.225	3
FP3	1	0.625	8
FP4	1	0.3	8
FP5	1	0.34	12







$$\mathcal{F} = \sigma_{NN}^{\text{in-medium}}/\sigma_{NN}^{\text{free}}$$

Q. Li, et. al., J. Phys. G 32, 407 (2006).

Q. Li, *et al.*, Phys. Rev. C **83**, 044617 (2011), Y. Wang, *et al.*, Phys. Rev. C **89**, 034606 (2014), P. Li, *et al.*, Phys. Rev. C **97**, 044620 (2018), O. Lopez, *et al.*, Phys. Rev. C **90**, 064602 (2014).

In-medium effects on nucleon-nucleon elastic cross section

- **Collective flows**

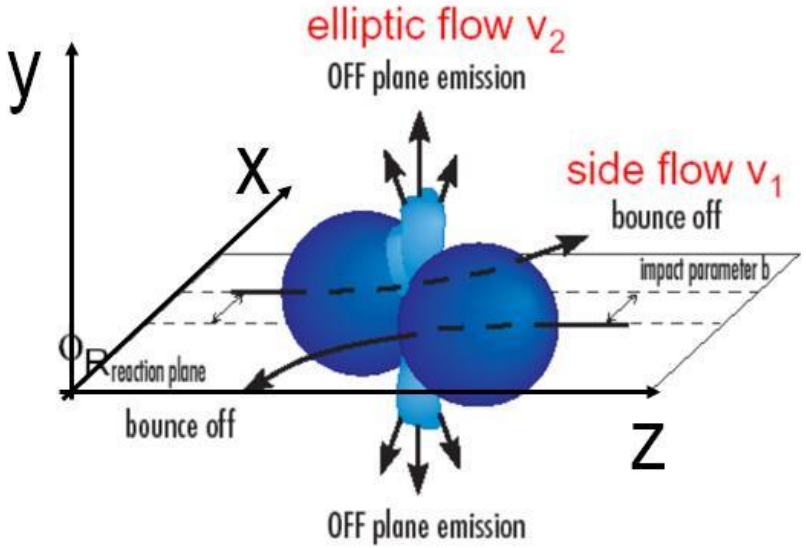
$$E\frac{d^3N}{d^3p} = \frac{1}{2\pi}\frac{d^2N}{p_t dp_t dy}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos[n(\phi - \Psi_{RP})]\right]$$

Direct flow:

$$v_1 \equiv \langle cos(\phi)\rangle = \left\langle \frac{p_x}{\sqrt{p_x^2 + p_y^2}} \right\rangle$$

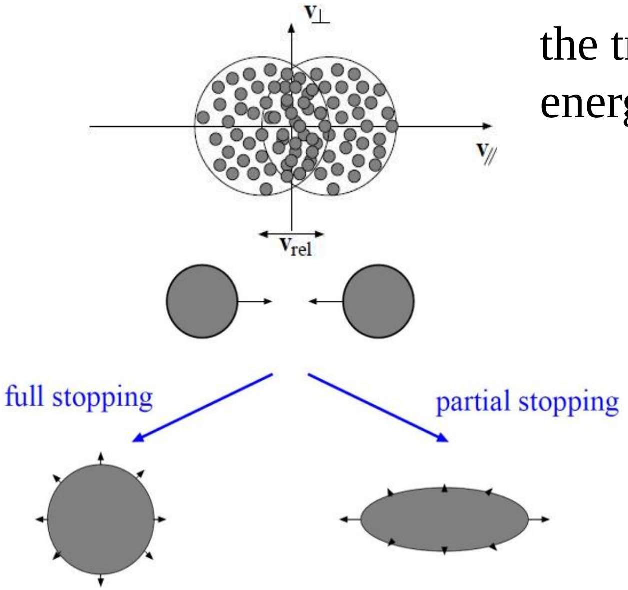
Elliptic flow:

$$v_2 \equiv \langle cos(2\phi)\rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

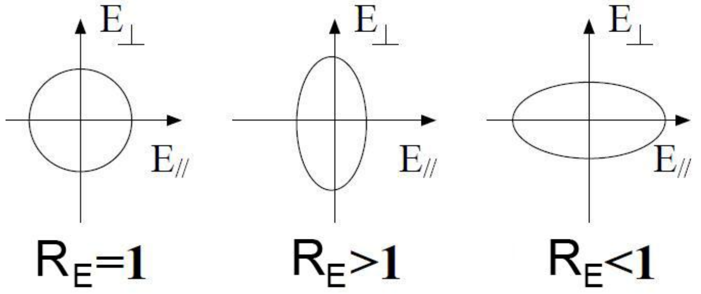


- **Nuclear stopping:**

$$R_E = \frac{1}{2}\frac{\sum_i^N E_i^{\perp}}{\sum_i^N E_i^{\parallel}},$$



the transverse and longitudinal c.m. energies for particle *i*



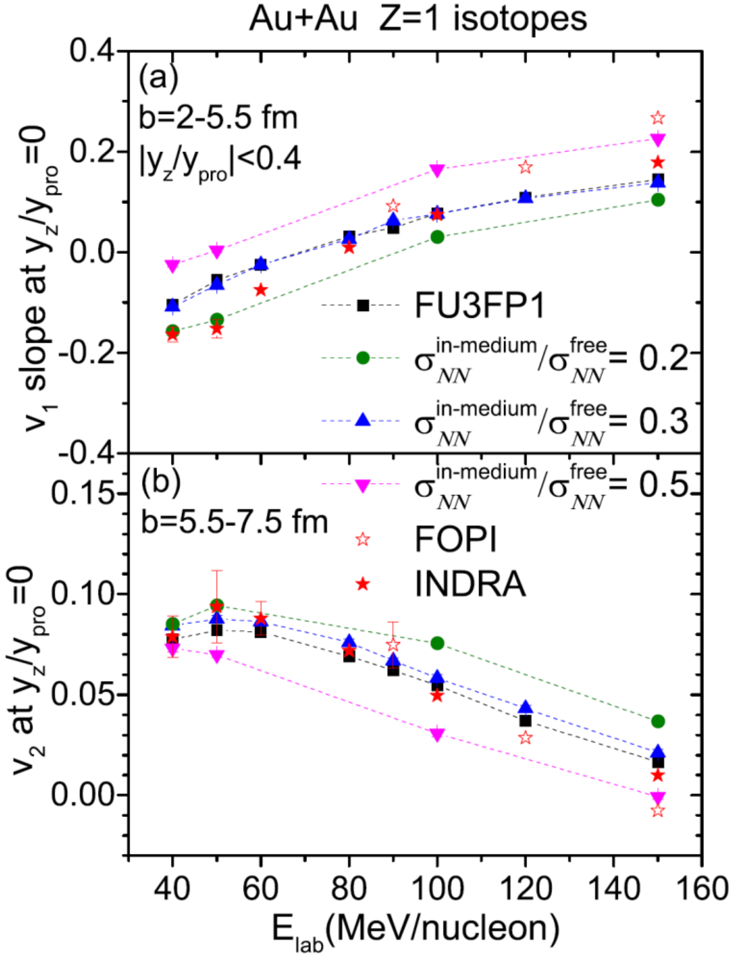
R_E=1: an isotropic emission,

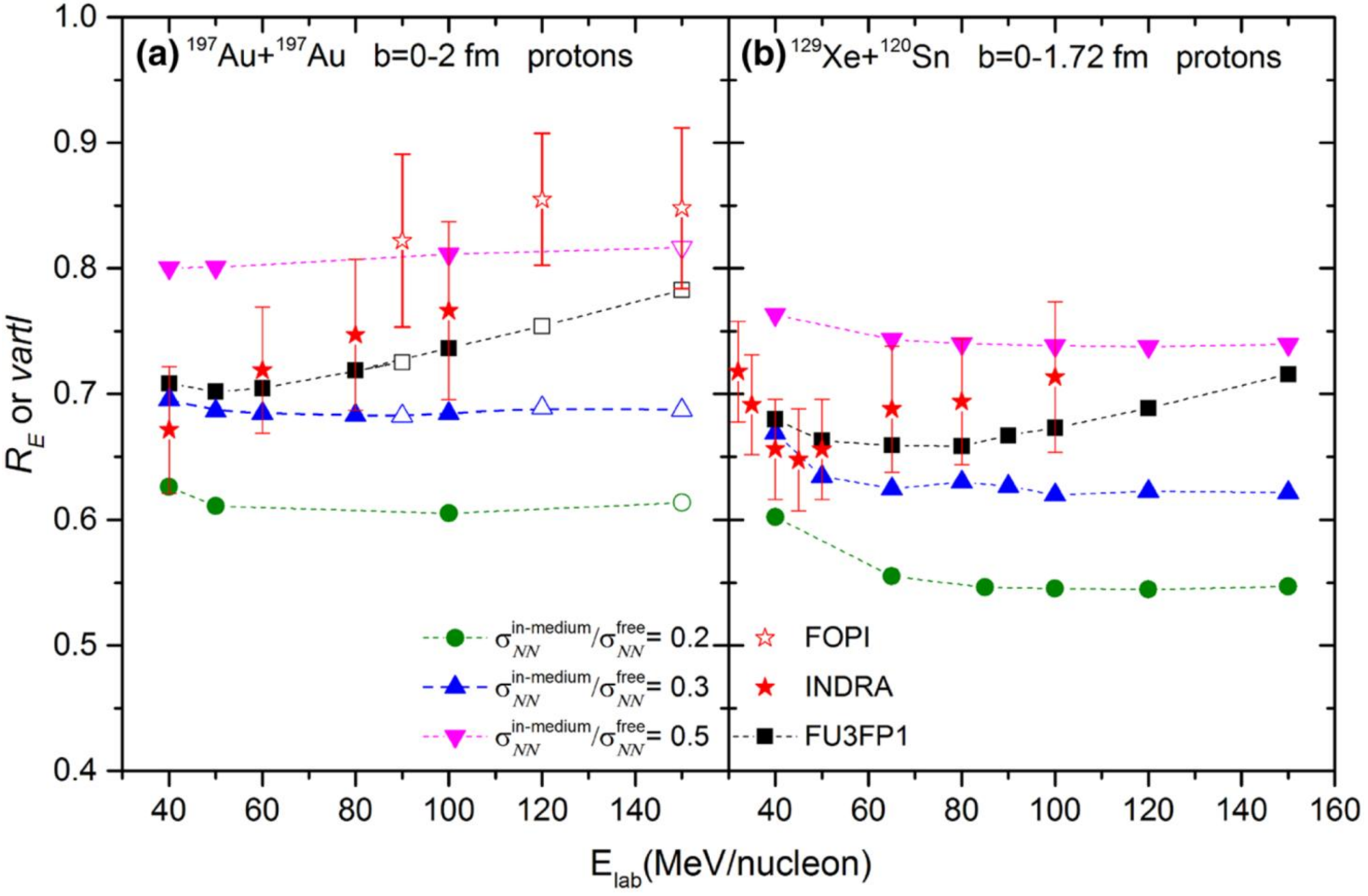
R_E<1: an elongated emission along the longitudinal direction given by the beam direction,

R_E>1: preferential emission in the plane transverse to the beam direction

INDRA Collaboration, Phys. Rev. Lett. 104, 232701 (2010), Phys. Rev. C 90, 064602 (2014) .

In-medium effects on nucleon-nucleon elastic cross section





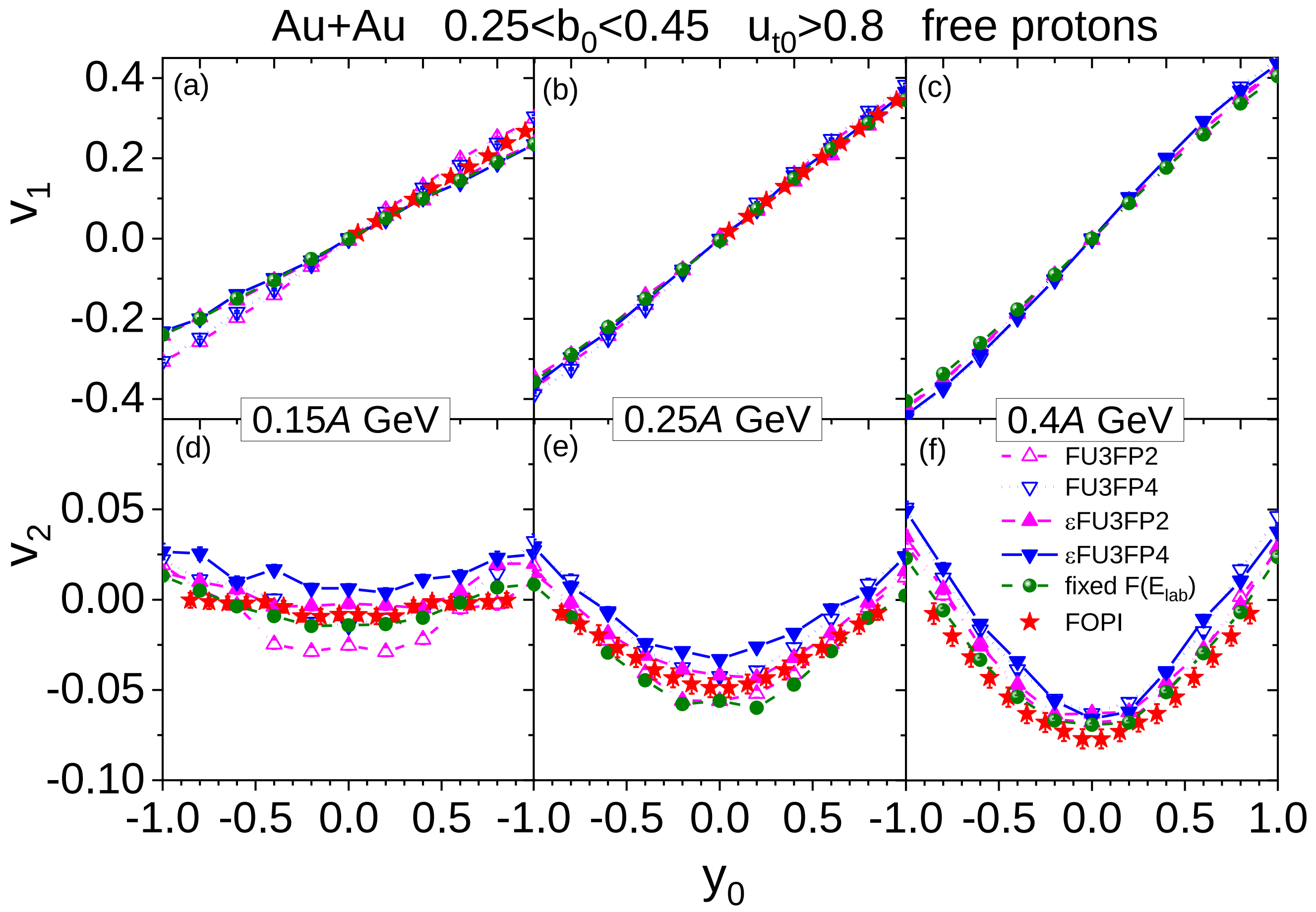
- With a larger *F* (means a smaller reduction on the cross section), we obtained a larger slope for v_1 and more negative v_2. Since the increased *NN* collision increases the flow effect.
- The v_1 and v_2 obtained with the FU3FP1 and *F*= 0.3 are very close to each other in the investigated energy region.
- At 40 MeV/nucleon, the experimental data can be well reproduced with *F*= 0.2, at 150 MeV/nucleon, calculations with *F*= 0.5 approach the data.

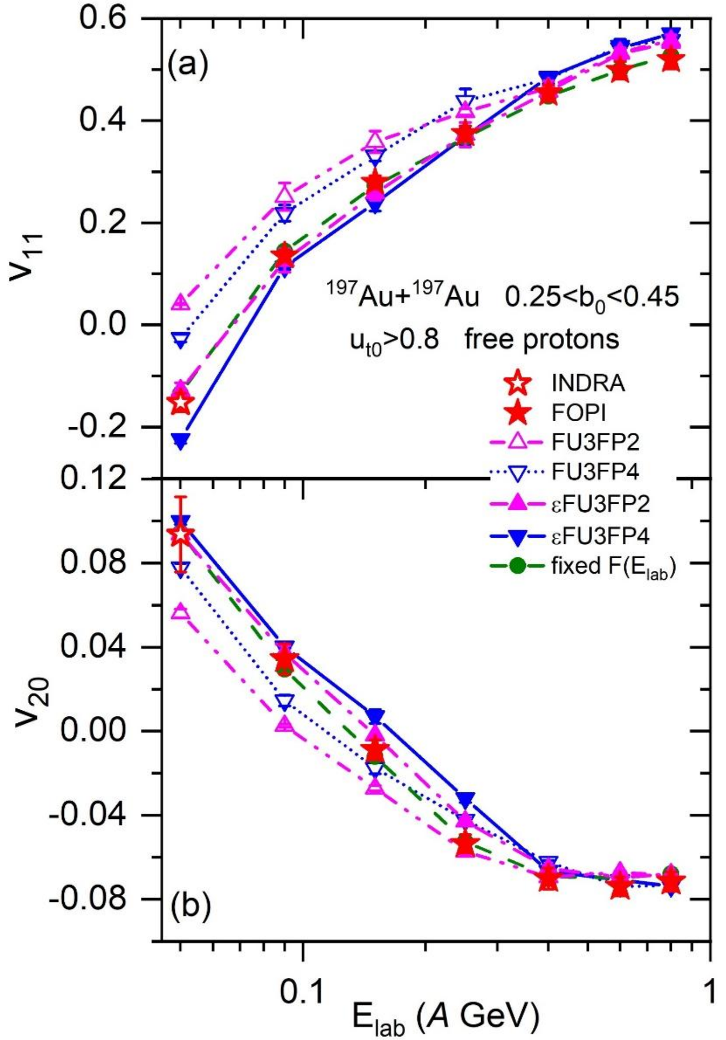
P. Li, *et al.*, Phys. Rev. C 97, 044620 (2018).

In-medium effects on nucleon-nucleon elastic cross section

Energy dependence

$$\varepsilon FUFP = tanh(E_{lab}/\varepsilon)FUFP, \varepsilon = 0.2$$

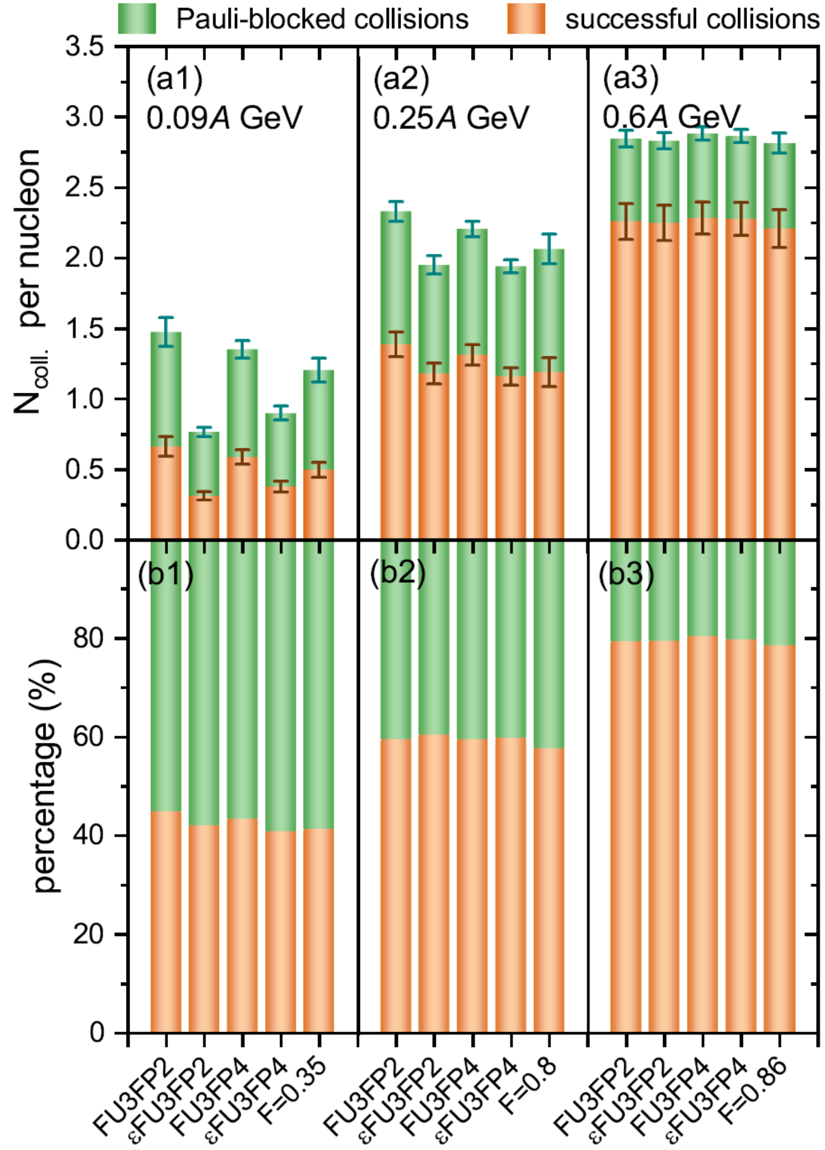




- FUFP and εFUFP sets are separated at low beam energies, and the difference vanishes at high energies. With considering the beam energy dependence of $\sigma_{el-NN}^{in-med.}$, the values of v_{11} (v_{20}) at mid-rapidity decrease (increase) at low energies, and these effects are weak at relatively high energies.

In-medium effects on nucleon-nucleon elastic cross section

Energy dependence



- The total and successful collisions number from simulations with FUFP sets are larger than that of εFUFP sets, these differences are almost vanishes at 0.6A GeV.

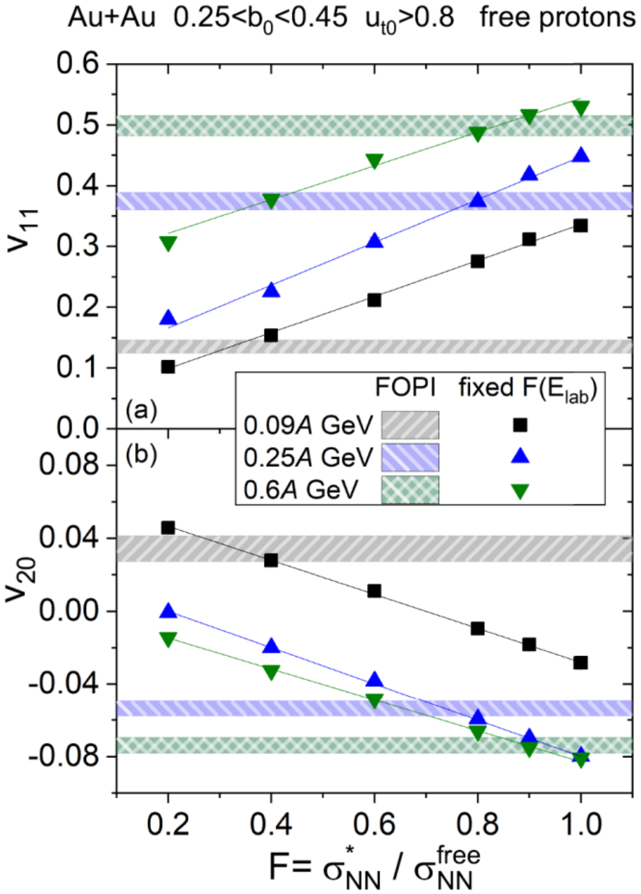
- Without considering the beam energy dependence on the $\sigma_{NN}^{in-med.}$, the collision number will increases, nucleons are more likely to undergo a bounce-off motion (follow a squeeze-out pattern), which reflects that the value of the v_{11} (v_{22}) at mid-rapidity increases (decreases).

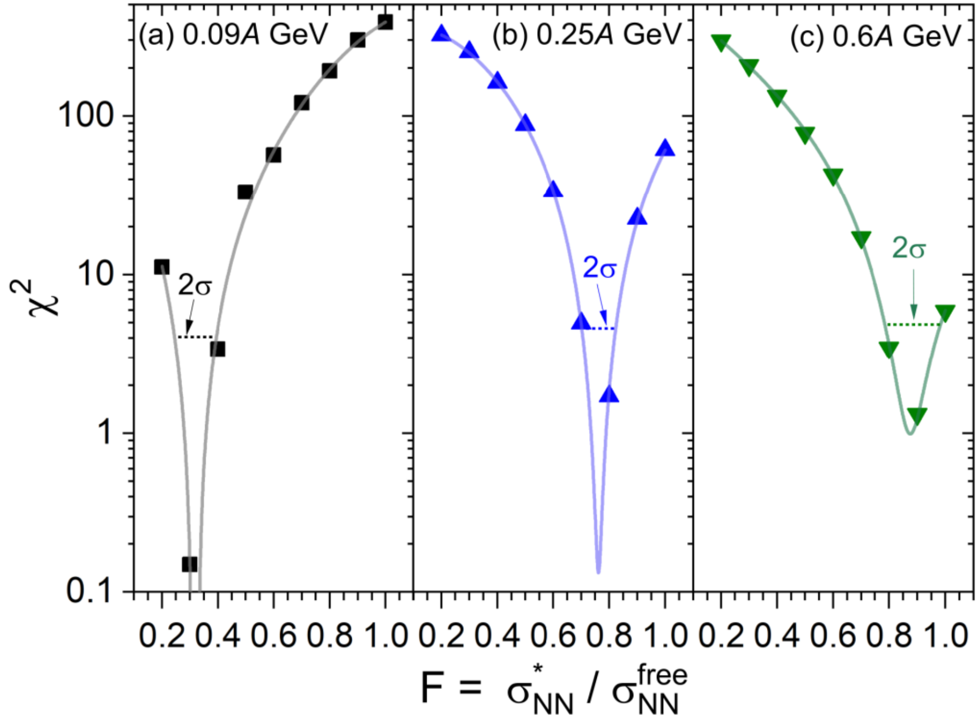
- With increasing the equivalent F, i.e., decreasing the in-medium effect, the collision number will increases.

- The percentage of the successful (Pauli-blocked) collision number to total collisions number hardly change when the in-medium correction factors are modified, since the Pauli blocking algorithms is not modified.

In-medium effects on nucleon-nucleon elastic cross section

Energy dependence





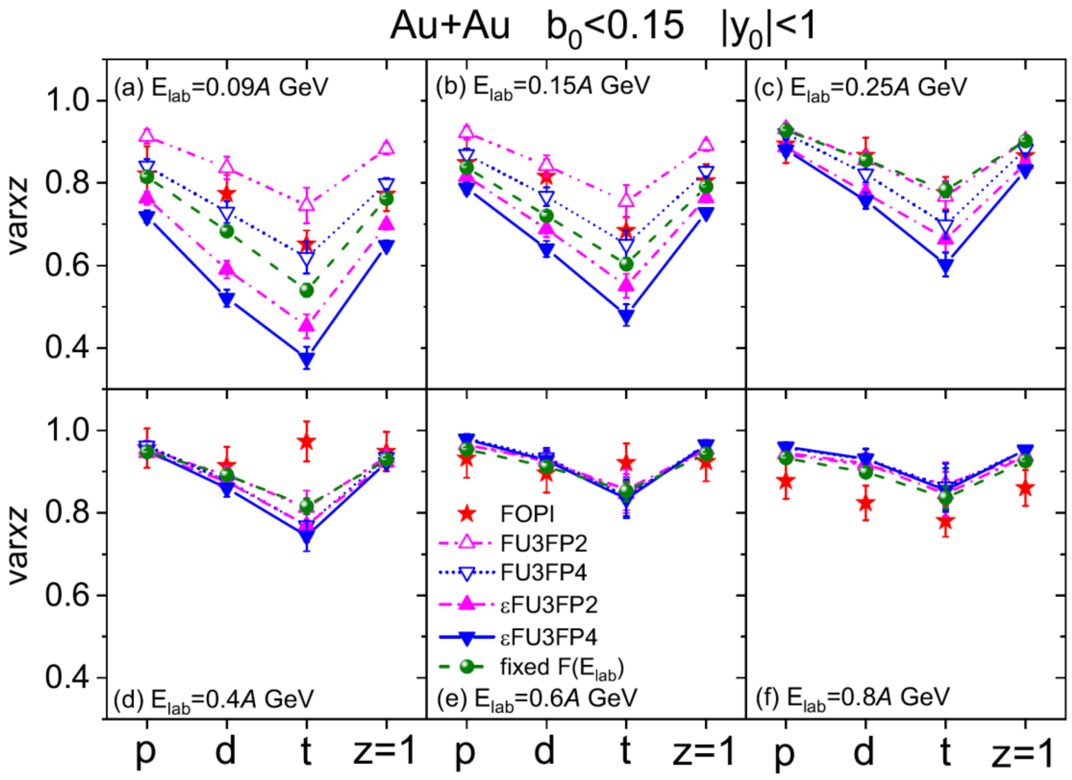
E_{lab} (A GeV)	0.09	0.15	0.25	0.4	0.6	0.8
fixed F	0.32	0.57	0.76	0.86	0.87	0.87
error	±0.07	±0.06	±0.06	±0.07	±0.10	±0.09

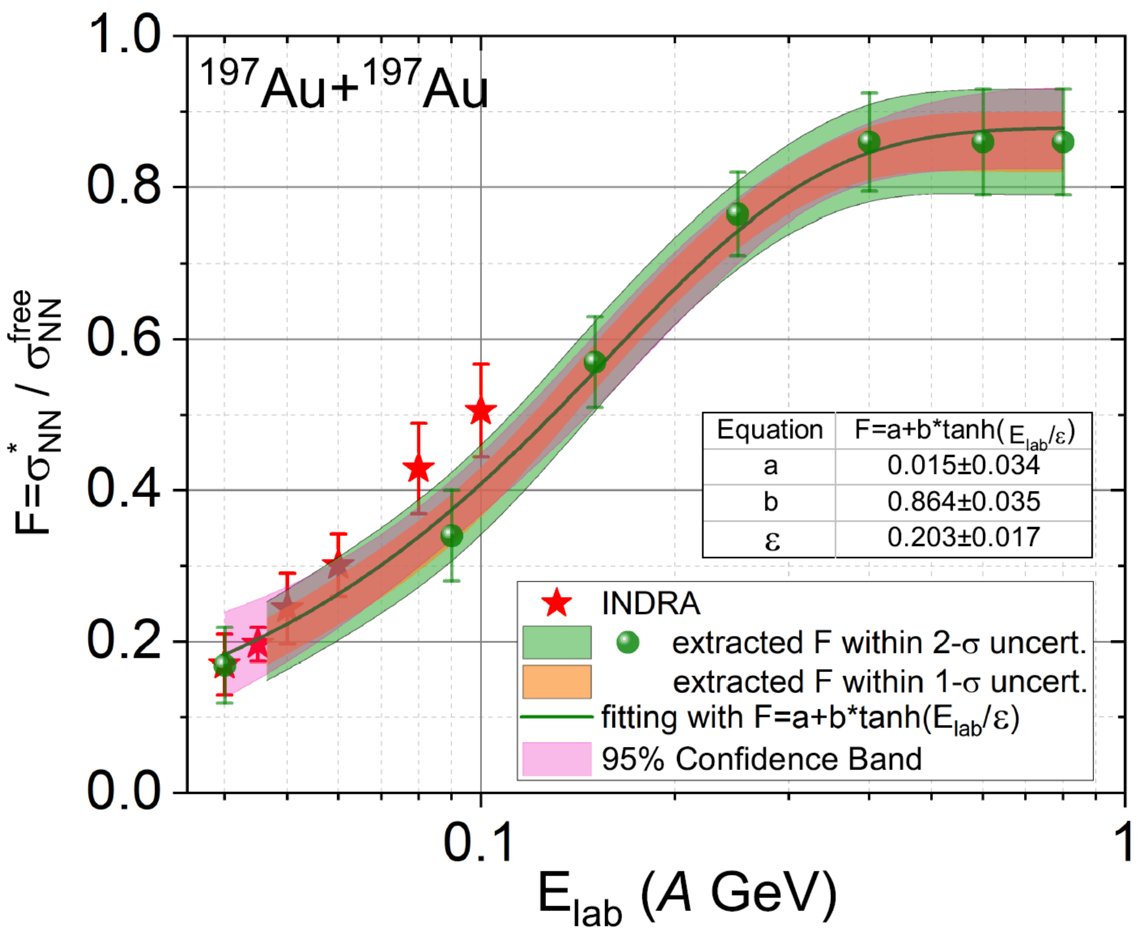
- A fairly well linear relationship between the collective flows and $\mathcal{F}$ can be seen, confirming that the collective flows are indeed sensitive to the in-medium effects.
- The extracted fixed equivalent $\mathcal{F}$ with a 2-σ confidence limit (at 95% confidence level) are shown in Table.

P. Li, *et al.*, Phys. Lett. B 828, 137019(2022).

In-medium effects on *NNECS*

Energy dependence



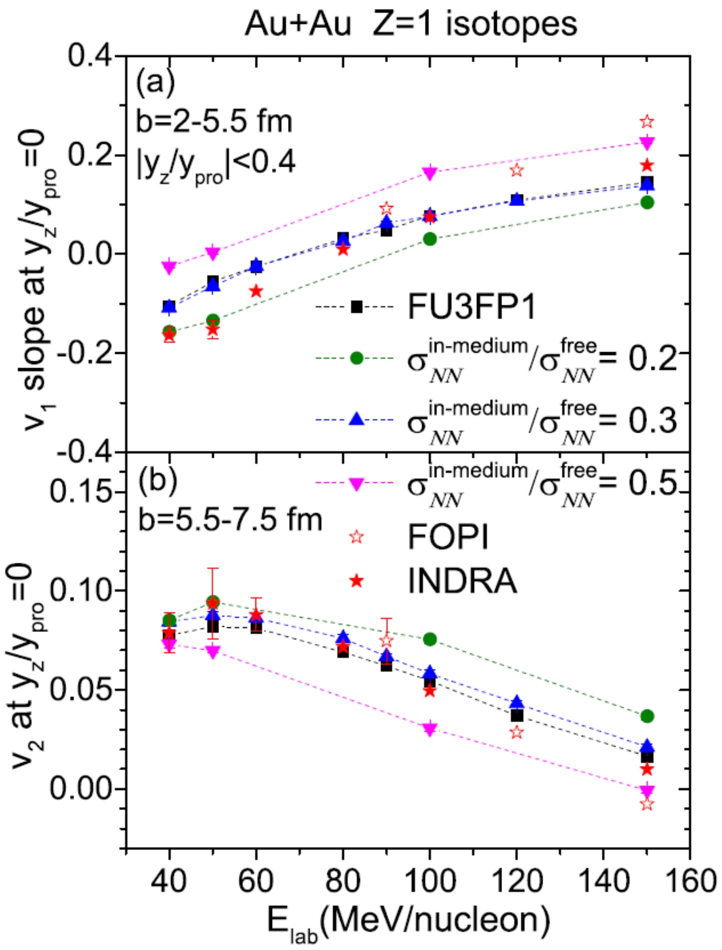


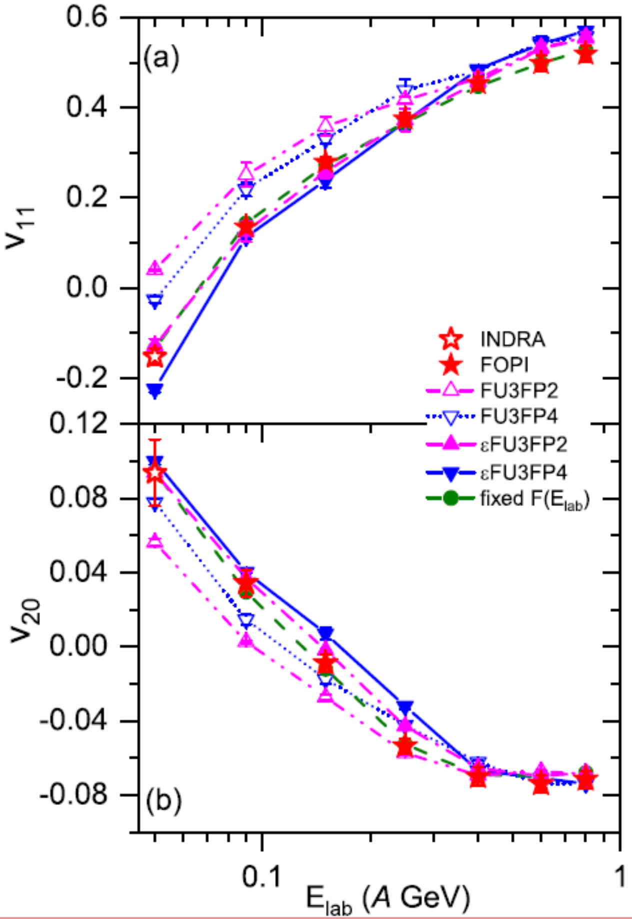
- The differences between the results from calculations with FUFP sets and εFUFP decrease with increasing E_{lab}. This beam energy dependence of the in-medium correction on *NNECS* becomes gradually weaker.
- Fitting with $\mathcal{F} = a + b\ \tan\left(\frac{E_{lab}}{\varepsilon}\right)$. It definitely provides intuitive and quantitative comprehension of in-medium effects on *NNECS*.

P. Li, *et al.*, Phys. Lett. B 828, 137019(2022).

In-medium effects on nucleon-nucleon elastic cross section

Energy dependence





$$\mathcal{F}(\rho,p)=\begin{cases} f_0, & p_{NN}>1\ \mathrm{GeV}/c,\\ \frac{\lambda+(1-\lambda)e^{-\frac{\rho}{\zeta\rho_0}}-f_0}{1+(p_{NN}/p_0)^{\kappa}}+f_0, & p_{NN}\leq 1\ \mathrm{GeV}/c.\end{cases}$$

$\Rightarrow$

$$\mathcal{F}(\rho,p)=\begin{cases} f_0, & p_{NN}>1\ \mathrm{GeV}/c,\\ \tanh(\frac{E_{lab}}{\varepsilon})[\frac{\lambda+(1-\lambda)e^{-\frac{\rho}{\zeta\rho_0}}-f_0}{1+(p_{NN}/p_0)^{\kappa}}+f_0], & p_{NN}\leq 1\ \mathrm{GeV}/c.\end{cases}$$

P. Li et.al., Phys. Rev. C 97, 044620 (2018). Nucl. Sci. Tech. 29, 177 (2018). Phys. Lett. B 828, 137019(2022).

In-medium nucleon-Delta elastic cross section

The RBUU equation of Δ(1232) distribution function read as;

$$\{p_\mu[\partial_x^\mu - \partial_x^\mu \Sigma_\Delta^\nu(x)\partial_\nu^p + \partial_x^\nu \Sigma_\Delta^\mu(x)\partial_\nu^p] + m_\Delta^* \partial_x^\nu \Sigma_\Delta^S(x)\partial_\nu^p\} \frac{f_\Delta(\mathbf{x},\mathbf{p},\tau)}{E_\Delta^*(p)} = C^\Delta(x,p), \quad (1)$$

Mean field:

σ, ω, ρ

$$\begin{aligned} L_I = & g_{NN}^\sigma \bar{\Psi}\Psi\sigma - g_{NN}^\omega \bar{\Psi}\gamma_\mu\Psi\omega^\mu - g_{NN}^\rho \bar{\Psi}\gamma_\mu \boldsymbol{\tau}\cdot\Psi\rho^\mu \\ & + g_{\Delta\Delta}^\sigma \bar{\Psi}_\Delta\Psi_\Delta\sigma - g_{\Delta\Delta}^\omega \bar{\Psi}_\Delta\gamma_\mu\Psi_\Delta\omega^\mu \\ & - g_{\Delta\Delta}^\rho \bar{\Psi}_\Delta\gamma_\mu\boldsymbol{\tau}\cdot\Psi_\Delta\rho^\mu . \end{aligned} \quad (2)$$

$$\begin{aligned} m_N^*(x) &= m_N + \Sigma_H^s(x), \\ m_\Delta^*(x) &= m_\Delta + \Sigma_H^s(x). \end{aligned} \quad (3)$$

The effective mass result is consistent with the relativistic mean field calculation result.

Collision part:

$$\begin{aligned} C^\Delta(x,p) = & \frac{1}{4}\int \frac{d\mathbf{p}_2}{(2\pi)^3}\int \frac{d\mathbf{p}_3}{(2\pi)^3}\int \frac{d\mathbf{p}_1}{(2\pi)^3} \\ & \times (2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)W^\Delta(p_1,p_2,p_3,p_4)[F_2-F_1]. \end{aligned} \quad (4)$$

The relationship between scattering cross section and transition probability:

$$\begin{aligned} \int v\frac{d\sigma^*}{d\Omega}d\Omega = & \int \frac{d\mathbf{p}_3}{(2\pi)^3}\int \frac{d\mathbf{p}_4}{(2\pi)^3}(2\pi)^4\delta^4(p+p_2-p_3-p_4) \\ & \times W^\Delta(p,p_2,p_3,p_4), \end{aligned} \quad (5)$$

The collision term read as:

$$C^\Delta(x,p) = \frac{1}{4}\int \frac{d\mathbf{p}_2}{(2\pi)^3}\sigma^\Delta(s,t)\nu_\Delta[F_2-F_1]\,d\Omega. \quad (6)$$

Transition probability

$$W^\Delta(p,p_2,p_3,p_4) = G(p,p_2,p_3,p_4) + p_3 \leftrightarrow p_4, \quad (7)$$

The G read as

$$G = \frac{g_{\Delta\Delta}^I g_{\Delta\Delta}^J g_{NN}^I g_{NN}^J T_c \Phi_c}{16E_\Delta^*(p)E^*(p_2)E_\Delta^*(p_3)E^*(p_4)}. \quad (8)$$

T_e is the isospin matrix,

$$T_e = \langle T|T_I|T_4\rangle\langle T_4|T_J|T\rangle\langle t_6|\tau_J|t_5\rangle\langle t_5|\tau_I|t_6\rangle, \quad (9)$$

In-medium nucleon-Delta elastic cross section

For isospin matrix Φ_e,

$$\begin{aligned}\Phi_e = \; & \mathrm{tr}\{\gamma_A\left(\not{p}_3 + m_\Delta^*\right) D^{\nu\mu}(p_3) \\ & \gamma_B \,\mathrm{tr}\left[\gamma_{B'}\left(\not{p}_2 + m^*\right)\gamma_{A'}\left(\not{p}_4 + m^*\right)\right] \\ & (\not{p} + m_\Delta^*) D_{\mu\nu}(p) D_{AA'} D_{BB'}\} \\ & \frac{1}{(p-p_3)^2 - m_I^2}\frac{1}{(p-p_3)^2 - m_J^2},\end{aligned} \tag{10}$$

where, $D_{\mu\nu}(p)$:

$$D_{\mu\nu}(p) = g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3m_\Delta}(\gamma_\mu p_\nu - \gamma_\nu p_\mu) - \frac{2}{3m_\Delta^2}p_\mu p_\nu. \tag{11}$$

The differential cross section of individual channel is

$$\begin{aligned}\int v\frac{d\sigma^*}{d\Omega}d\Omega = & \int\frac{d\mathbf{p}_3}{(2\pi)^3}\int\frac{d\mathbf{p}_4}{(2\pi)^3}(2\pi)^4\delta^4(p+p_2-p_3-p_4) \\ & \times W^\Delta\left(p, p_2, p_3, p_4\right),\end{aligned} \tag{12}$$

d_i is isoapin matrice, D_i are spin matrix. A_i is coupling constant：

$$\begin{aligned} & A_1 = {g_{NN}^\sigma}^2 {g_{\Delta\Delta}^\sigma}^2, && A_2 = {g_{NN}^\omega}^2 {g_{\Delta\Delta}^\omega}^2, \\ & A_3 = g_{NN}^\sigma g_{\Delta\Delta}^\sigma g_{NN}^\omega g_{\Delta\Delta}^\omega, && A_4 = {g_{NN}^\rho}^2 {g_{\Delta\Delta}^\rho}^2, \\ & A_5 = g_{NN}^\sigma g_{\Delta\Delta}^\sigma g_{NN}^\rho g_{\Delta\Delta}^\rho, && A_6 = g_{NN}^\omega g_{\Delta\Delta}^\omega g_{NN}^\rho g_{\Delta\Delta}^\rho 。\end{aligned} \tag{13}$$

For total cross section.

$$\sigma_{N\Delta\to N\Delta}^* = \frac{1}{8}\int d\Omega\frac{d\sigma_{N\Delta\to N\Delta}^*}{d\Omega}. \tag{14}$$

In addition, Mandelstam variables have the following relationship

$$\begin{aligned} & s = (p_1+p_2)^2 - \left[E_\Delta^*(p) - E^*(p_2)\right]^2 - (\mathbf{p}+\mathbf{p}_2)^2, \\ & t = (p_1-p_3)^2 = m_\Delta^{*2} + m^{*2} - \frac{1}{2s}\left[s^2 - \left(m_\Delta^{*2} - m^{*2}\right)^2\right], \\ & + 2\,|\mathbf{p}|\,|\mathbf{p}_3|\cos\theta, \\ & u = (p_1-p_4)^2 = 2m_\Delta^{*2} + 2m^{*2} - s - t.\end{aligned} \tag{15}$$

Momentum

$$|\mathbf{p}| = |\mathbf{p}_3| = \frac{1}{2\sqrt{s}}\sqrt{(s - m^{*2} - m_\Delta^{*2})^2 - 4m^{*2}m_\Delta^{*2}}, \tag{16}$$

On shell condiditon,

$$p_{i,i=1-4}^2 = m_{i,i=1-4}^{*2}. \tag{17}$$

The vertex effective form factor due to finite size of nucleons and short-range correlation effects

$$F_{NNM}(t) = \frac{\Lambda^2}{\Lambda^2 - t}, \tag{18}$$

The cut off mass of σ、ω、ρ

$$\Lambda_\omega = 1200\mathrm{MeV}, \Lambda_\omega = 808\mathrm{MeV}, \Lambda_\rho = 800\mathrm{MeV}. \tag{19}$$

For：$\Delta - \Delta$ −meson vertex：$\Lambda_\Delta = 0.4\Lambda.$

Density dependence of nucleon-Delta elastic cross section

The interaction part of effective Lagrangian density

$$L_I = g_{NN}^{\sigma}\bar{\Psi}\Psi\sigma - g_{NN}^{\omega}\bar{\Psi}\gamma_\mu\Psi\omega^\mu - g_{NN}^{\rho}\bar{\Psi}\gamma_\mu\boldsymbol{\tau}\cdot\Psi\rho^\mu$$
$$+ g_{\Delta\Delta}^{\sigma}\bar{\Psi}_\Delta\Psi_\Delta\sigma - g_{\Delta\Delta}^{\omega}\bar{\Psi}_\Delta\gamma_\mu\Psi_\Delta\omega^\mu$$
$$- g_{\Delta\Delta}^{\rho}\bar{\Psi}_\Delta\gamma_\mu\boldsymbol{\tau}\cdot\Psi_\Delta\rho^\mu.$$

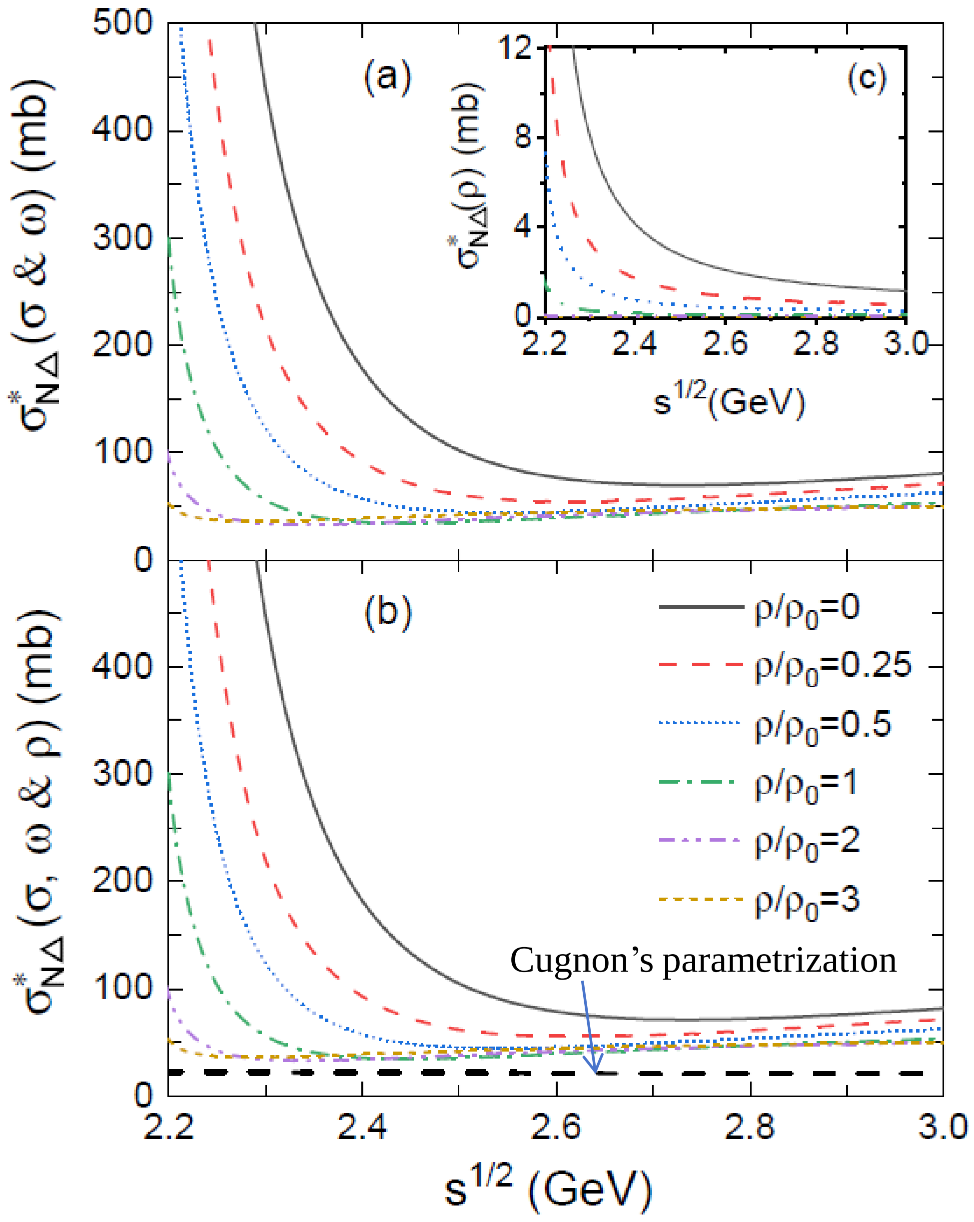
$$g_{NN}^{i}(\rho) = g_i(\rho_{sat})f_i(\xi), \quad i = \sigma, \omega, \quad f_i(\xi) = a_i\frac{1+b_i(\xi+d_i)^2}{1+c_i(\xi+d_i)^2}, \quad \xi = \frac{\rho}{\rho_{sat}}.$$

$$g_{NN}^{\rho}(\rho) = g_\rho(\rho_{sat})e^{-a_\rho(\xi-1)}.$$

$$\frac{g_{\Delta\Delta}^{\sigma}}{g_{NN}^{\sigma}} = 1.0, \quad \frac{g_{\Delta\Delta}^{\omega}}{g_{NN}^{\omega}} = 0.8, \quad \frac{g_{\Delta\Delta}^{\rho}}{g_{NN}^{\rho}} = 0.7;$$

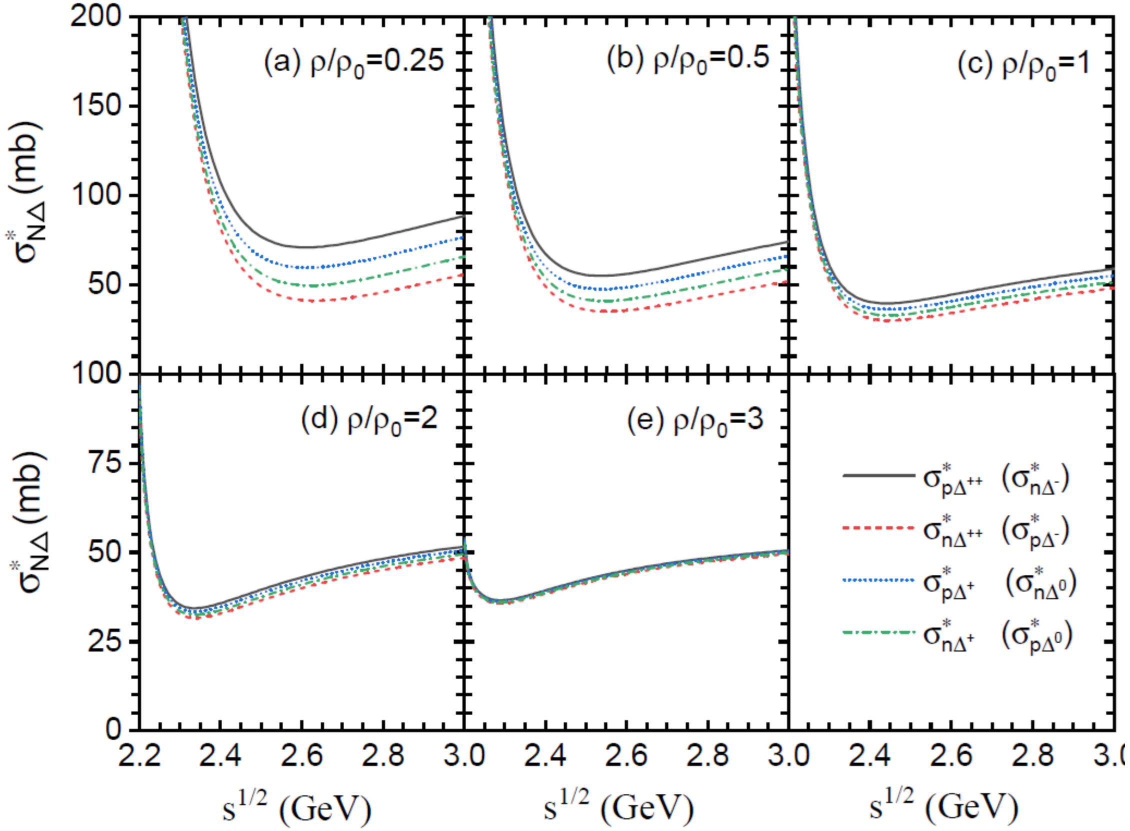
A. R. Raduta, Phys. Lett. B 814, 136070 (2021).
G. A. Lalazissis, T. Niksic, D. Vretenar, and P. Ring, Phys. Rev. C 71, 024312 (2005).

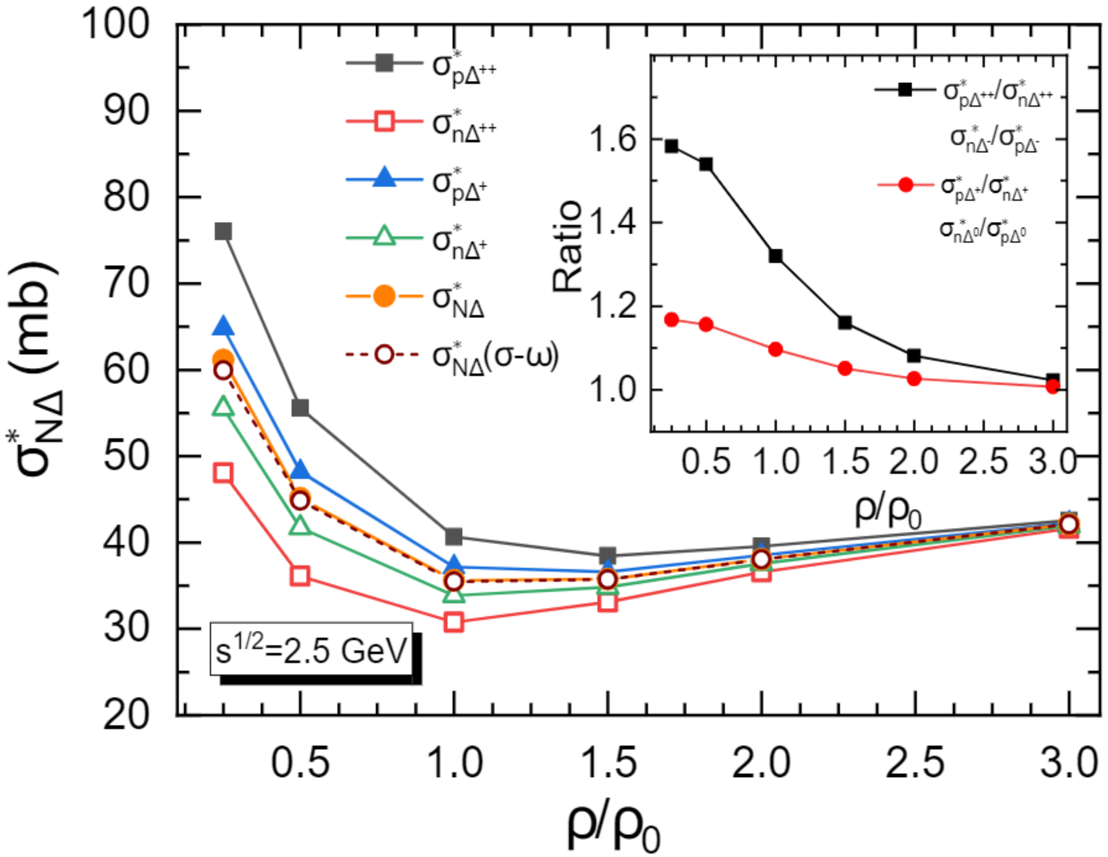
- Decreases with increasing energy, decrease with increasing density, shows an significant suppressed effect in the low-density region.
- The contribution of ρ meson exchange is minimal, its contribution approaches zero at 2 to 3 times saturation density.



M. Nan, P. Li, Y. Wang, W. Zuo, Eur. Phys. J. A 60:131(2024).

Density dependence of nucleon-Delta elastic cross section

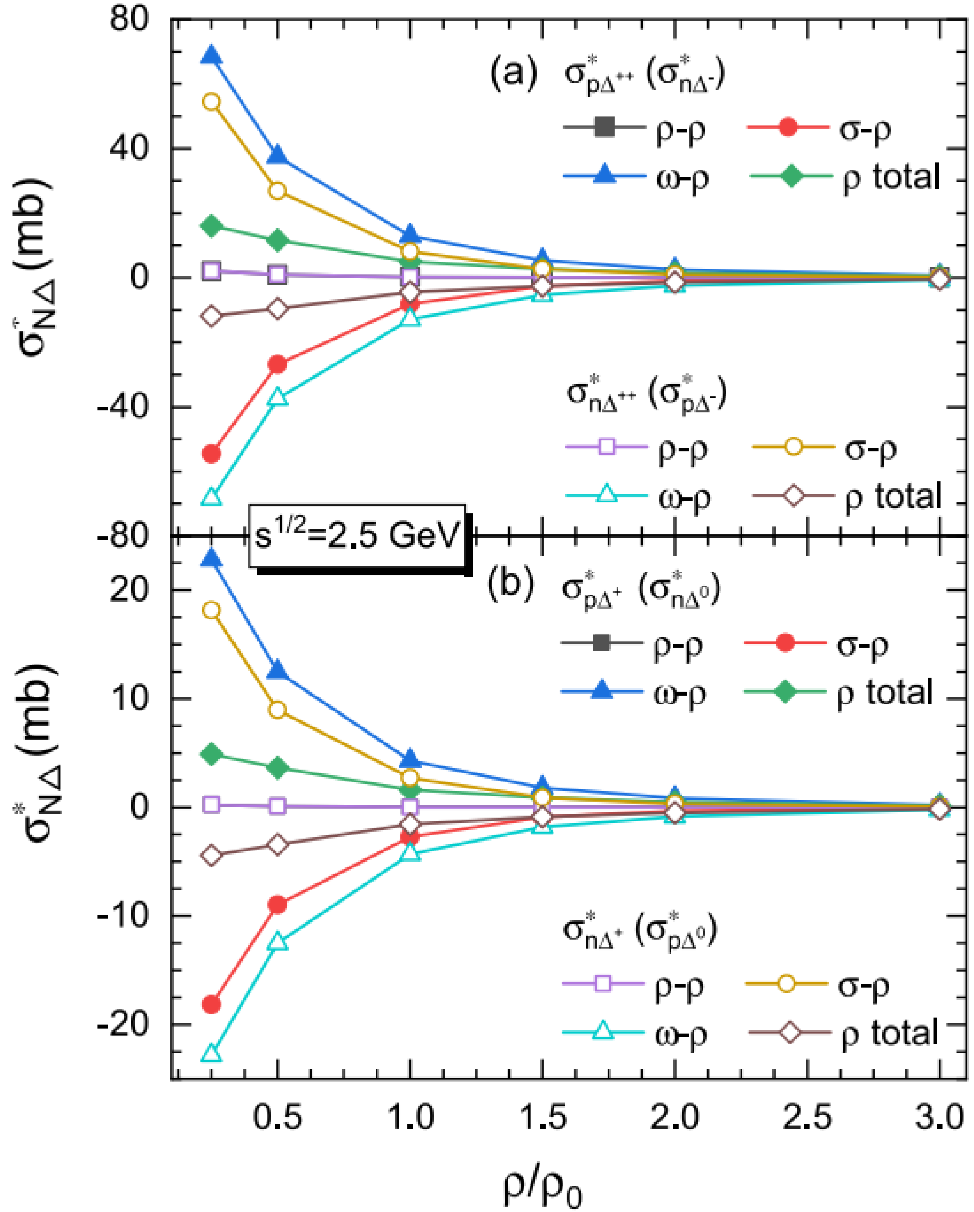




- The introduction of the ρ meson field in the effective lagrangian leads to significant isospin effects in the individual cross section.
- The splitting effects in $\sigma^*_{N\Delta}$ between individual channels, caused by the isospin effect, decrease with increasing reduced density.
- The isospin effect between different isospin-separated channels weakens as the energy and/or density increases, and when the density reaches 3ρ0, the isospin effect almost disappears.

Density dependence of nucleon-Delta elastic cross section

Contributions from the ρ-meson related exchange terms:



- All individual $\sigma^*_{N\Delta}$ decreases with density and eventually approaches zero, which is results from the density-dependent properties of the baryon-baryon-meson coupling constants.

- The contribution of meson exchange terms: ω-ρ > σ-ρ > ρ-ρ, which means the contributions of the ω-ρ and σ-ρ terms are primarily determined by the σ and ω meson fields

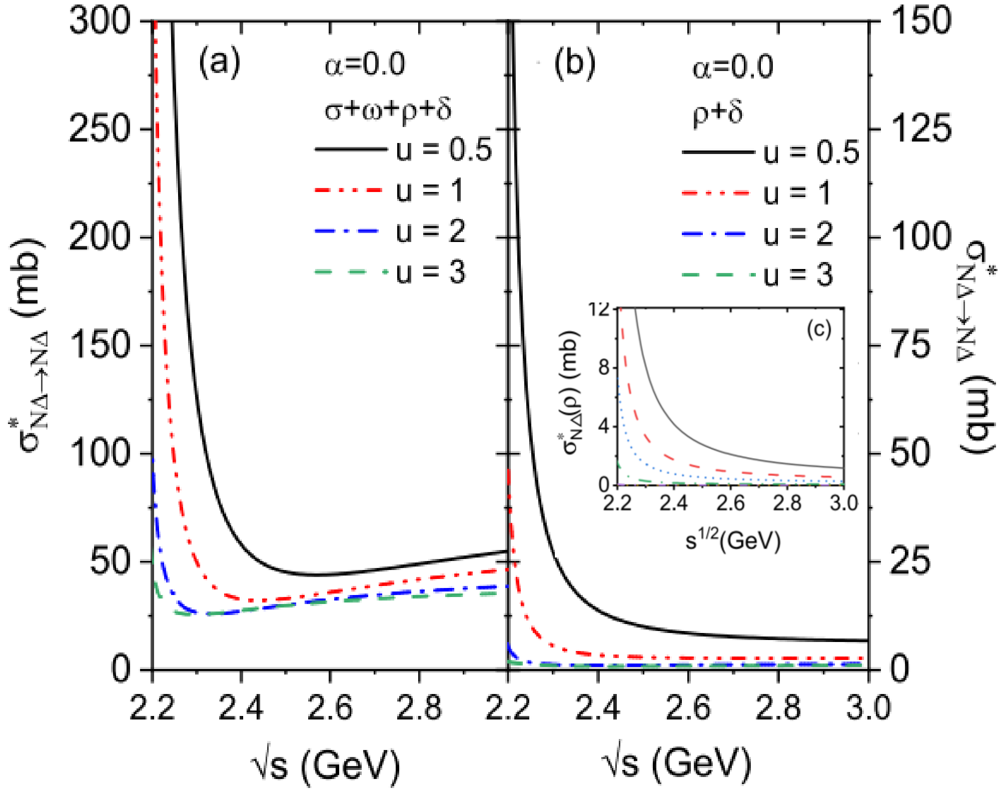
- Due to the cancellation between the contributions from different exchange terms, the $\sigma^*_{N\Delta}$ which includes the total contributions from ρ meson exchanges has a weak density dependence.

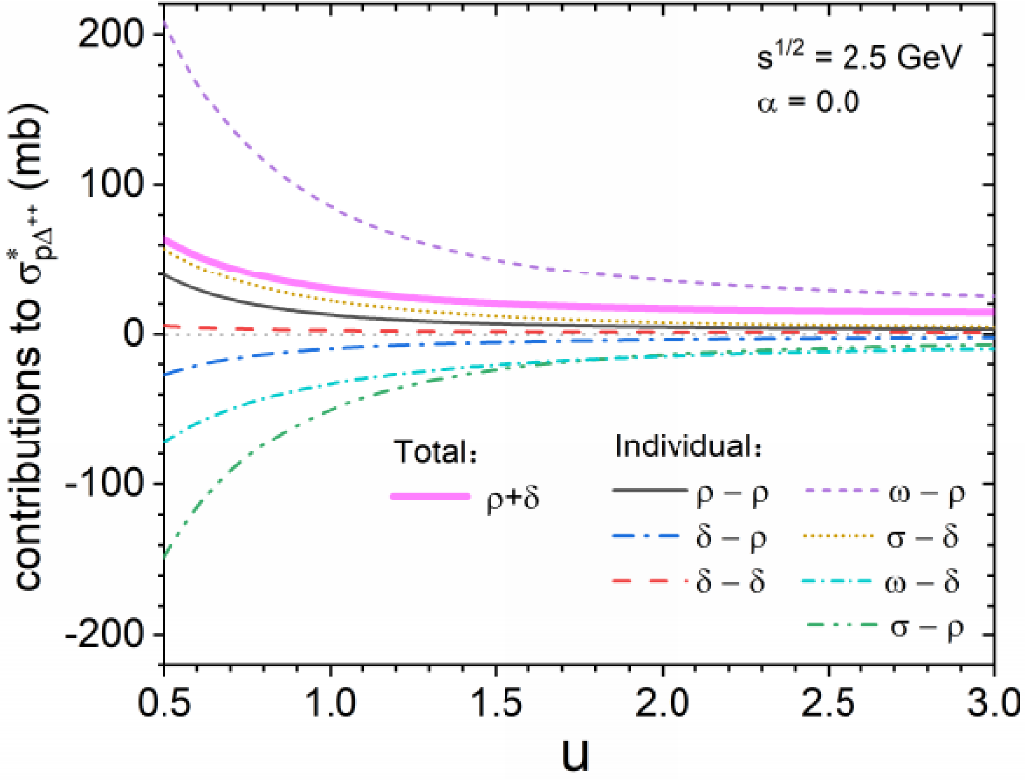
- The contributions from isovector-isovector meson (ρ-ρ) exchanges are positive, while for isoscalar-isovector meson (σ-ρ and ω-ρ) exchanges, the contributions are always opposite.

M. Nan, P. Li, Y. Wang, W. Zuo, Eur. Phys. J. A 60:131(2024).

Isospin dependence of nucleon-Delta elastic cross section

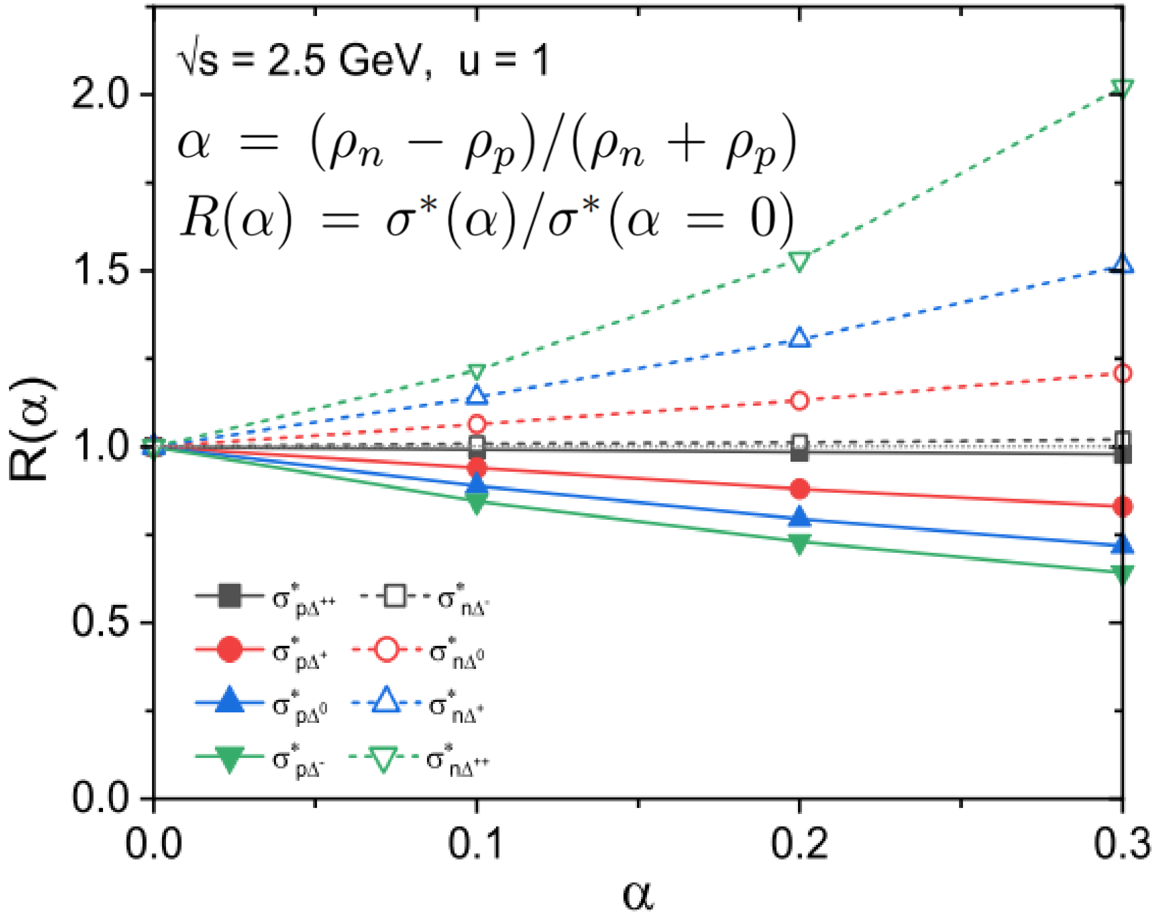
The δ meson field is further considered beside the σ, ω, and ρ meson fields.

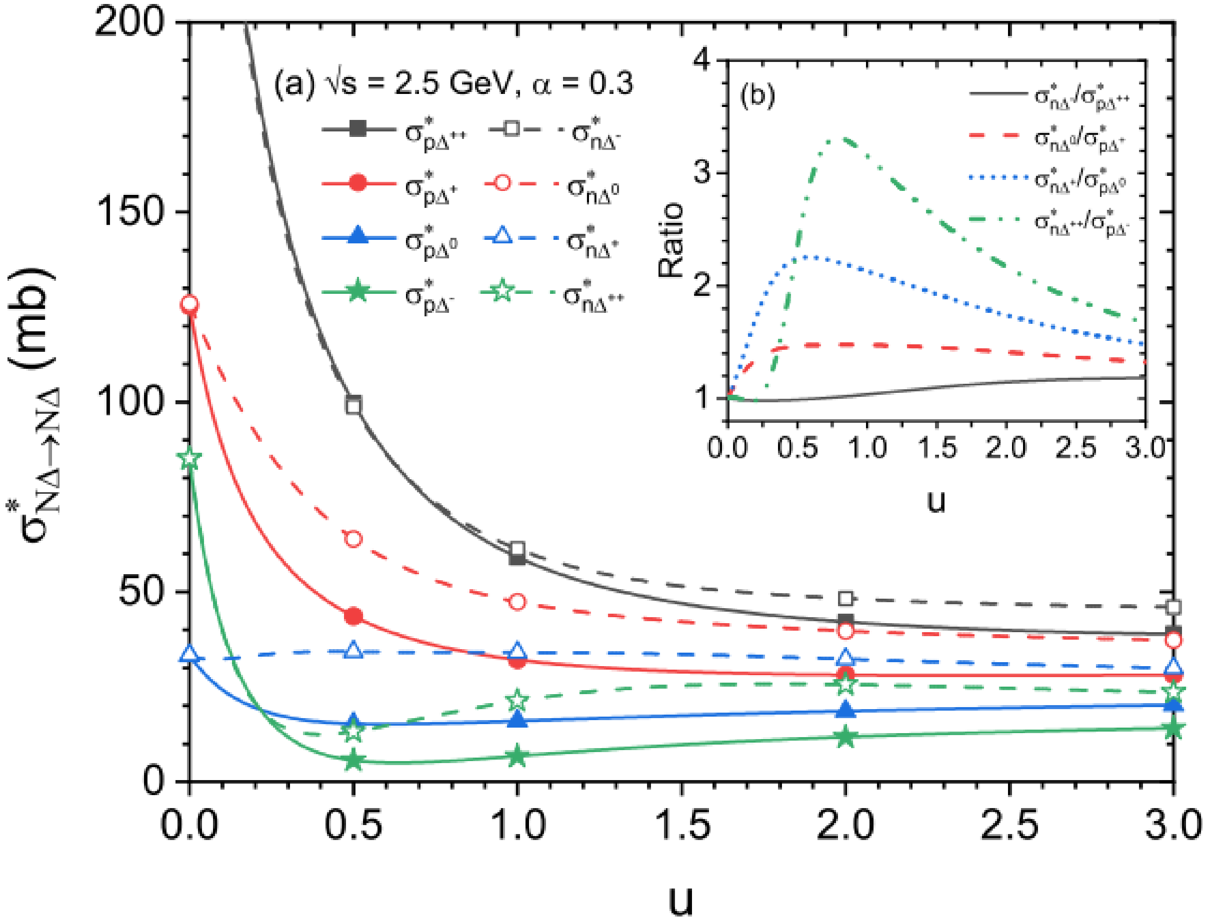




- $\sigma^*_{N\Delta}$ decreases with increasing density, indicating a visible density dependent suppression of nuclear medium.
- The ρ and δ meson related-terms have a larger contribution than that of ρ meson field.
- The contribution of each meson exchange term decreases with increasing reduced density, the baryon-baryon-meson coupling constants and the effective masses of nucleons and Δ particles.
- Obvious cancellation effect, but the net-contribution of ρ and δ related exchange terms to the $\sigma^*_{p\Delta^{++}}$ is larger than 0.

Isospin dependence of nucleon-Delta elastic cross section





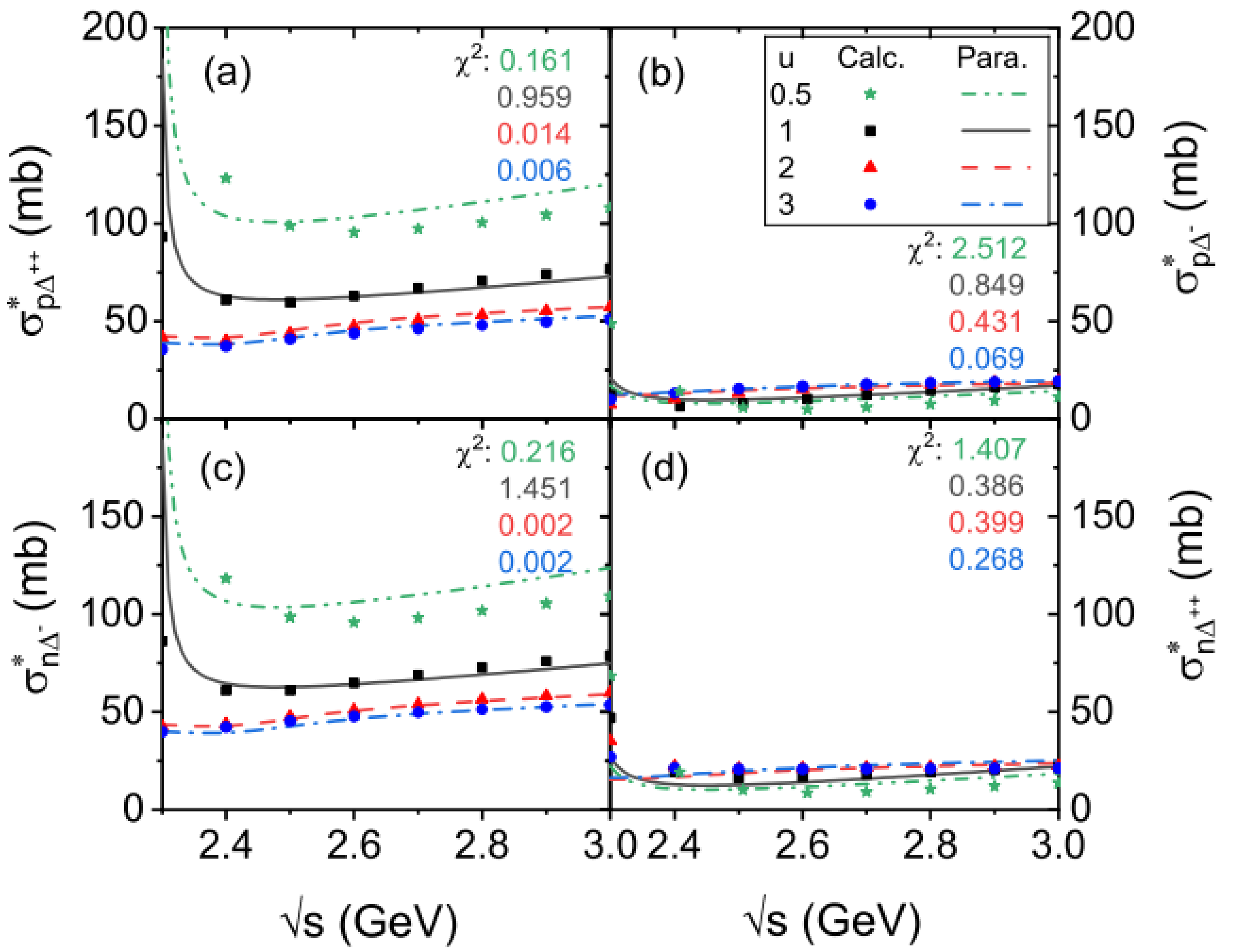
- $R(\alpha)$ for pΔ channels is decreased, while that for nΔ channels is increased as α increases from 0.0 to 0.3, since the contribution of δ meson exchange to the effective masses of protons, neutrons and Δ-isobars have opposite signs.
- The isospin effect, which introduced by isovector ρ and δ meson fields, in NΔ → NΔ channel should not be negligible even at such a high energy and density.

M. Nan, P. Li, Q. Li, W. Zuo, submitted.

Energy-, density- and isospin-dependent in-medium nucleon-Delta elastic cross section

based on the above calculations within the RBUU theoretical framework, a parametrization for the energy (√s)-, density (u)-, and isospin (α)-dependent NΔ elastic cross section is proposed,

$$\sigma^*_{N\Delta\to N\Delta}(\sqrt{s},u,\alpha) = \left[g\left(\sqrt{s}+h\right) + \frac{i\left(\sqrt{s}+j\right)}{k+\left(\sqrt{s}+l\right)^2}\right] \times \left[a+bu+c\exp(du)\right] \times \left(1+e\alpha+f\alpha^2\right),$$



The parametrization can well reproduce the microscopic calculation results within the c.m. energy region of 2.3 ≤√s ≤ 3 GeV and the density range 0.5 ≤ u ≤ 3, which indicates that the proposed formula provides a reliable description of cross section within a wide range of energy, density, as well isospin asymmetry, and can serve as a trustworthy input for transport model simulations of HICs.

Summary

- ✓ The in-medium NN/ND elastic cross section is suppressed when compared to the free one.
- ✓ The in-medium correction effect on the NN/ND elastic cross section is energy-, density-, and isospin-dependent.

- The in-medium effects of the NN elastic cross section decreases with increasing beam energy (~80% at E_{lab} = 0.04A GeV and ~24% at 0.25A GeV).
- A phenomenological formula for in-medium NN elastic cross section is presented by fitting the extracted F, and this formula can be easily incorporated in transport model.

- ➢ Both ρ and δ meson related exchange terms have non-negligible contributions to $\sigma^*_{N\Delta}$.
- ➢ The isospin effect introduced by the isovector ρ and δ meson fields still has an unignorable effect on the individual NΔ elastic cross section even at 2-3ρ_0
- ➢ A reliable parameterized formula of the energy-, density-, and isospin-dependent NΔ cross section is proposed.

Thank you for your attention.